Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting

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It is often suggested that requiring juries to reach a unanimous verdict reduces the probability of convicting an innocent defendant while increasing the probability of acquitting a guilty defendant. We construct a model that demonstrates how strategic voting by jurors undermines this basic intuition. We show that the unanimity rule may lead to a high probability of both kinds of error and that the probability of convicting an innocent defendant may actually increase *with the size of the jury. Finally, we demonstrate that a wide variety of voting rules, including simple majority rule, lead to much lower probabilities of both kinds of error.*

According to Lord Devlin, "Trial by jury is not an instrument of getting at the truth; it is a process designed to make it as sure as possible that no innocent man is convicted" (Klaven and Zeisel 1966, 190). It is commonly thought that requiring juries to reach a unanimous verdict is exactly the mechanism that protects innocent defendants and that this protection comes at the cost of an increased probability of acquitting a guilty defendant. We construct a model that demonstrates how strategic voting by jurors undermines this basic intuition. The unanimity rule may lead to a high probability of both errors, and the probability of convicting an innocent defendant may actually increase with the size of the jury. We also demonstrate that the unanimity rule is an exceptionally bad rule. A wide variety of voting rules, including simple majority, lead to much lower probabilities of both errors.

There is a large literature on juries and jury decision making (e.g., Adler 1994, Klaven and Zeisel 1966, Levine 1992, and McCart 1964). A central argument for juries, formalized in the literature on Condorcet's jury theorem, is that a group will make a better decision than an individual (Klevorick et al. 1984, Ladha 1992, Miller 1986, and Young 1988). In Condorcet's jury theorem it is assumed that jurors have private information about the guilt or innocence of the defendant. This captures the idea that jurors interpret evidence differently because of their different life experiences and competencies. Since a jury vote aggregates their private information, juries make fewer errors than any individual.

Until recently, the literature has assumed that each juror will behave as if her vote alone determines the outcome. Several recent articles have demonstrated that such behavior by jurors is frequently irrational and that the combination of private information and common interests creates an incentive for jurors to vote strategically (Austen-Smith and Banks 1996; Feddersen and Pesendorfer 1996, 1997a, 1997b; McLennan 1996; Myerson 1997; Wit 1996).[1] There is even experimental research that finds empirical support for the strategic voting hypothesis (Ladha, Miller and Oppenheimer 1996). The incentive to vote strategically arises because a juror's vote only matters when a vote is pivotal and because the information possessed by other jurors is relevant for a juror's decision. For example, under the unanimity rule, a vote is pivotal only if all the other jurors have voted to convict. The fact that all other jurors have voted to convict reveals additional information about the guilt of the defendant. Such information may overwhelm the juror's private assessment of the case and cause a juror otherwise inclined to vote for acquittal to vote for conviction instead.

In this article we examine the implications of strategic voting by jurors and demonstrate that basic intuitions about the consequences of requiring a unanimous vote may be dramatically wrong. We construct a model of jury decision making that incorporates private information and strategic voting. We show that the requirement of a unanimous verdict may actually result in a significantly higher probability of convicting an innocent defendant than would, for example, simple majority rule. We conclude with a few brief remarks on the implications of our results for jury reform.

THE MODEL

There are n jurors, $j = 1, \ldots, n$, who must decide the fate of a defendant. The defendant is either guilty (G) or innocent (I).[2] We assume that G and I occur with equal probability.[3]

Jurors are uncertain whether the defendant is guilty or innocent. We assume that each juror gets a signal g or i that is correlated with the true state. Specifically, we assume that

$$\Pr(g|G) = \Pr(i|I) = p.$$

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[1] Jurors have common interests to the extent that all prefer to convict a guilty defendant and acquit an innocent defendant.

[2] Feddersen and Pesendorfer (1997a, 1997b) generalize the simple two-state model to multiple states. Adding additional states will not fundamentally alter our results.

[3] This assumption can be easily relaxed without significantly changing our results.

Hence, the parameter $p \in (.5, 1)$ is the probability that a juror receives the "correct" signal (g if the defendant is guilty, i if the defendant is innocent), and $1 - p$ is the probability that the juror receives the "incorrect" signal (i if the defendant is guilty, g if the defendant is innocent).[4]

We assume the signal is private information. Since jurors observe the same facts at the trial and engage in deliberations prior to taking the final vote, the assumption may seem inappropriate. Yet, there are several reasons the complete disclosure of private information through the deliberation process may not occur. For example, some jurors may have technical knowledge that is relevant for the decision but that cannot be fully communicated in the limited amount of time available. Furthermore, while all jurors agree that they prefer convicting the guilty and acquitting the innocent, each may have a different threshold of reasonable doubt.[5] Even such minimal preference diversity may create incentives for jurors not to reveal their private information in deliberations. For example, a juror predisposed to convict may be reluctant to reveal her innocent signal lest another juror with a higher threshold who received a guilty signal vote to acquit. Since we do not model the effect of jury deliberations, determining that effect from a theoretical standpoint is beyond the scope of this article.

The jury reaches a decision by taking a simultaneous vote. Each juror must vote either to convict or to acquit. If the number of votes to convict is larger than or equal to $\hat{k}$, then the defendant is convicted; otherwise, the defendant is acquitted. The number $\hat{k}$ defines the voting rule used by the jury. Thus, if $\hat{k} = n$, then a unanimous verdict is required to convict; if $\hat{k} = (n + 1)/2$, then a conviction is obtained by a simple majority vote. There are two possible outcomes of the jury's vote: either the defendant is convicted (C) or acquitted (A).

Given the voting rule $\hat{k}$, juror j's behavior can be described by the strategy σ_j: {g, i} $\rightarrow [0, 1]$, which maps the set of signals into a probability of voting to convict.

We assume all jurors have preferences, given by $u(A, I) = u(C, G) = 0$, $u(C, I) = -q$, and $u(A, G) = -(1 - q)$, where $q \in (0, 1)$. Thus, if a guilty defendant is convicted or an innocent defendant is acquitted, then each juror's payoff is zero. If an innocent defendant is convicted, then the juror's payoff is $-q$; if a guilty defendant is acquitted, then the juror's payoff is $-(1 - q)$. The parameter q exactly characterizes a juror's threshold of reasonable doubt. A juror who believes the defendant is guilty with probability higher than q will prefer the defendant to be convicted. The larger the value of q, the less concern jurors have for acquitting a guilty defendant relative to convicting an innocent defendant. We assume that jurors employ the same standard of reasonable doubt, that is, q is identical for all jurors. This assumption is made for technical convenience. Below, we indicate how our results generalize to the case in which jurors' preferences are represented by different values of q. (See Feddersen and Pesendorfer 1996, 1997a, 1997b and Coughlan 1997 for examples of strategic voting under preference diversity.)

Let $\beta(k, n)$ denote the posterior probability that the defendant is guilty, conditional on observing n signals, k of which are guilty:

$$\beta(k, n) = \frac{p^k(1-p)^{n-k}}{p^k(1-p)^{n-k} + (1-p)^k p^{n-k}}.$$

If $\beta(k, n) > q$, then, given all the information available to the jury, the defendant is guilty beyond a reasonable doubt. Therefore, the optimal outcome from the jurors' point of view is to convict. Similarly, if $\beta(k, n) < q$, then the optimal outcome for the jurors is to acquit. We assume that there is a k^* with $n \geq k^* \geq 1$, such that

$$\beta(k^* - 1, n) < q < \beta(k^*, n).$$

This assumption implies, on the one hand, that if the jurors know they all have received the guilty signal, then they always want to convict the defendant. On the other hand, if the jurors believe they all have received an innocent signal, then they always want to acquit.

As a baseline for comparison we first consider nonstrategic voting. We say that a juror votes informatively if she votes guilty when she receives a guilty signal and innocent when she receives an innocent signal. If jurors vote informatively, then the unanimity rule ($\hat{k} = n$) leads to a lower probability of convicting an innocent defendant than would any other rule ($\hat{k} < n$). The probability an innocent defendant is convicted under the unanimity rule given informative voting is $(1 - p)^n$. It is easy to see that this probability is smaller than the probability of convicting an innocent defendant under any other rule.[6] Under the unanimity rule, the probability a convicted defendant is guilty is given by

$$\frac{p^n}{p^n + (1-p)^n}.$$

This probability converges to one as the size of the jury grows, that is, $n \rightarrow \infty$. Conversely, the probability of acquitting a guilty defendant is strictly higher under the unanimity rule than under any other rule.[7]

As has been shown in the literature cited above, informative voting does not typically constitute equi-

[4] The assumption that the probability of receiving signal i in state I is identical to the probability of receiving signal g in state G can be relaxed without significantly changing our results.

[5] Support for the hypothesis of differing thresholds of reasonable doubt can be found in Klaven and Ziesel (1966, chap. 14). They show that when the verdict the judge would have chosen differs from the verdict chosen by the jury, it is more frequently the case that the jury acquits when the judge would have convicted than the other way around.

[6] If the rule requires $\hat{k}$ of n votes to be guilty, then under informative voting the probability of convicting an innocent defendant is $\sum_{j=\hat{k}}^{n} \binom{n}{j}(1-p)^j p^{n-j}$.

[7] This probability is $1 - p^n$ under the unanimity rule, which is strictly larger than $\sum_{j=0}^{\hat{k}} \binom{n}{j}(1-p)^{n-j} p^j$, which is the corresponding probability for the rule that requires $\hat{k} < n$ guilty votes.

librium behavior. In the following we analyze Nash equilibria of the described voting game. This allows us to compare the performance of the various voting rules.

THE UNANIMITY RULE

In this section we examine Nash equilibria under the unanimity rule, that is, $\hat{k} = n$.

Consider the case in which $k^* = n$, that is, $\beta(n-1, n) \leq q < \beta(n, n)$. In this case informative voting is an equilibrium under the unanimity rule. To see this, suppose that all jurors vote to convict if and only if they receive the signal g. If a juror is pivotal and receives the signal i, then he knows that $n - 1$ of n jurors received the signal g. Therefore, he believes that the defendant is guilty with probability $\beta(n-1, n) \leq q$, that is, not guilty beyond a reasonable doubt, and hence he (weakly) prefers that the defendant be acquitted. Thus, a vote to acquit is optimal. Conversely, if the juror receives the signal g, then he believes that the defendant is guilty with probability $\beta(n, n) > q$, and hence he prefers that the defendant be convicted.

Now consider the case in which

$$\beta(n-1, n) > q. \qquad \textbf{(1)}$$

This condition says that if a juror could observe all the signals, and if $n - 1$ of the n signals are g, then the juror prefers to convict the defendant. Note that, for any fixed q, there is an $\bar{n}$ such that condition 1 is true for any $n > \bar{n}$.[8]

Suppose that condition 1 is satisfied and a juror believes that the other jurors are voting informatively. A rational juror will condition her vote not only on her private information but also on what she believes others must know in the event her vote is pivotal. Under the unanimity rule a vote is pivotal only when all the other jurors have voted to convict. A juror who receives an innocent signal and believes that the other jurors are voting informatively *must* believe that the probability the defendant is guilty is exactly $\beta(n-1, n)$. But since $\beta(n-1, n) > q$, the juror will ignore her private signal and vote to convict. Therefore, if (1) holds, then informative voting cannot be a Nash equilibrium.

THE PROBABILITY A CONVICTED DEFENDANT IS INNOCENT

In contrast to the results under informative voting, strategic voting under the unanimity rule imposes a *lower* bound on the probability that a convicted defendant is innocent. Proposition 1 provides this bound for *any* Nash equilibrium and shows that the bound is independent of the size of the jury.

PROPOSITION 1. *Consider any Nash equilibrium in which the defendant is convicted with strictly positive probability. Then the probability that a convicted defendant is innocent is bounded below by*

$$\min\left\{1/2, \frac{1-q}{1+q\dfrac{2p-1}{(1-p)^2}}\right\}$$

Proof. See Appendix A. *Q.E.D.*

It is straightforward to extend proposition 1 to the case in which jurors have different thresholds of reasonable doubt. If we let q denote the maximal threshold of reasonable doubt among the jurors, then the bound given in proposition 1 continues to hold in the more general case in which each juror has a different threshold, q_j, $j = 1, \ldots, n$.

To understand the intuition behind proposition 1, it is useful to distinguish two cases. First consider the (trivial) case in which the defendant is always convicted. Then the probability that a convicted defendant is innocent is simply 1/2, that is, the prior probability that the defendant is innocent.

Second, consider the case in which the defendant sometimes is acquitted. In this case at least one juror votes to acquit with strictly positive probability when he receives the signal i. A juror will only vote to acquit if the fact that a vote is pivotal is not overwhelming evidence of guilt. To put this formally, let piv_j denote the event that juror j's vote is pivotal. Upon observing signal i, j can only vote to acquit in any Nash equilibrium if

$$\Pr(G|piv_j, i) \leq q.$$

In other words, the probability the defendant is guilty conditional on j's vote being pivotal and on her private signal i must be less than or equal to her threshold of reasonable doubt q.

Now observe that the only difference between the event a vote is pivotal and the event the defendant is convicted is one vote. The most information that one additional vote can reveal occurs when a pivotal voter votes informatively. It follows that conditional on a conviction the defendant must be innocent with probability bounded strictly away from zero and that this lower bound is independent of the size of the jury. This outcome represents a stark contrast with the results under the assumption of informative voting, in which the fraction of convicted defendants who are innocent goes to zero as the size of the jury increases.

CONVICTING INNOCENT DEFENDANTS

In this section we explicitly compute an equilibrium under the unanimity rule. We then demonstrate that in this equilibrium innocent defendants are convicted with strictly positive probability even when the jury size is very large. This contrasts with the result under informative voting, in which the probability of convicting any defendant goes to zero as the size of the jury grows.

For the subsequent analysis we examine *symmetric*

[8] To see this, observe that $\beta(n-1, n) = p^{n-1}(1-p)/(p^{n-1}(1-p) + (1-p)^{n-1}p) = 1/(1 + (1-p/p)^{n-1}p)$. Since $p > 1/2$, it follows that $\lim_{n\to\infty} \beta(n-1, n) = 1$.

Nash equilibria, in which all jurors who receive the same signal take the same (possibly mixed) action. We therefore drop the subscript identifying particular jurors from our notation and write $\sigma(s)$ to denote the probability a juror votes to convict upon observing signal s. A symmetric strategy profile is given by $(\sigma(i), \sigma(g))$.

Under any voting rule there is a Nash equilibrium in which all jurors vote the same way independent of their signal; for example, under the unanimity rule all jurors may vote to acquit independent of their private signal. Since no juror can influence the outcome, this is always an equilibrium.[9]

In the following we focus on profiles in which jurors change their vote as a function of their private information with positive probability. We call such profiles responsive. In order to formalize our definition of responsive profiles we first define

$$\gamma_G = \sigma(g)p + (1-p)\sigma(i)$$

as the probability that a juror votes to convict if the defendant is guilty and

$$\gamma_I = (1-p)\sigma(g) + p\sigma(i)$$

as the probability that a juror votes to convict if the defendant is innocent. We say that the profile $(\sigma(g), \sigma(i))$ is responsive if $\gamma_G \neq \gamma_I$.

If condition 1 holds, then informative voting is not an equilibrium, and any responsive symmetric equilibrium must be in mixed strategies. More precisely, each juror must both vote to convict and vote to acquit with positive probability whenever she receives a signal i, that is, $\sigma(i) > 0$. When the juror receives the signal g, she votes to convict with probability 1, that is, $\sigma(g) = 1$.[10]

For a mixed strategy profile to be an equilibrium, a juror who receives an innocent signal must be indifferent between voting to acquit and voting to convict. This occurs when, conditional on $n-1$ others voting guilty and the juror receiving signal i, the probability that the defendant is guilty is exactly equal to q. By Bayes's law we get the following equilibrium condition for the unanimity rule:

$$\frac{(1-p)(\gamma_G)^{n-1}}{(1-p)(\gamma_G)^{n-1} + p(\gamma_I)^{n-1}} = q. \qquad \textbf{(2)}$$

Therefore,

$$\sigma(i) = \frac{\left(\frac{(1-q)(1-p)}{qp}\right)^{1/(n-1)} p - (1-p)}{p - \left(\frac{(1-q)(1-p)}{qp}\right)^{1/(n-1)} (1-p)}. \qquad \textbf{(3)}$$

Since $\sigma(i)$ is a mixed strategy, it must be that $\sigma(i) \leq 1$. Furthermore, in a responsive equilibrium it must be the case that $\sigma(i) < 1$; otherwise, each juror votes to convict with probability one irrespective of the signal. Examining equation 3 we see that $\sigma(i) < 1$ as long as $q > 1 - p$. If $q \leq 1 - p$, then there does not exist a responsive equilibrium. Instead, there is an equilibrium where $\sigma(i) = 1$, which implies that the probability of convicting an innocent defendant is one, and the probability of acquitting a guilty defendant is zero.[11]

To understand why there cannot be a responsive equilibrium if $q < 1 - p$, note that in equilibrium a guilty vote of some other juror can never be information in favor of the innocence of the defendant. If a juror receives no information from being pivotal, then he believes that the defendant is guilty with probability $1 - p$ when he receives the signal i. Hence, conditional on his vote being pivotal, each juror must believe the defendant to be guilty with probability at least $1 - p$, and each juror must vote to convict even if he receives the signal i.

Using equation 3 it is straightforward to compute the probability of making each type of error in equilibrium. When $q \geq 1 - p$, the probability that an innocent defendant is convicted is given by

$$l_I(p, q, n) = (\gamma_I)^n$$
$$= \left(\frac{(2p-1)\left(\frac{(1-q)(1-p)}{qp}\right)^{1/(n-1)}}{p - (1-p)\left(\frac{(1-q)(1-p)}{qp}\right)^{1/(n-1)}}\right)^n,$$

and the probability of acquitting a guilty defendant is

$$l_G(p, q, n) = 1 - (\gamma_G)^n$$
$$= 1 - \left(\frac{(2p-1)}{p - (1-p)\left(\frac{(1-q)(1-p)}{qp}\right)^{1/(n-1)}}\right)^n.$$

Proposition 2 summarizes our findings in this section and demonstrates that the probability of convicting an innocent defendant stays bounded away from zero for all n. Similarly, the probability of acquitting a guilty defendant also stays bounded away from zero.

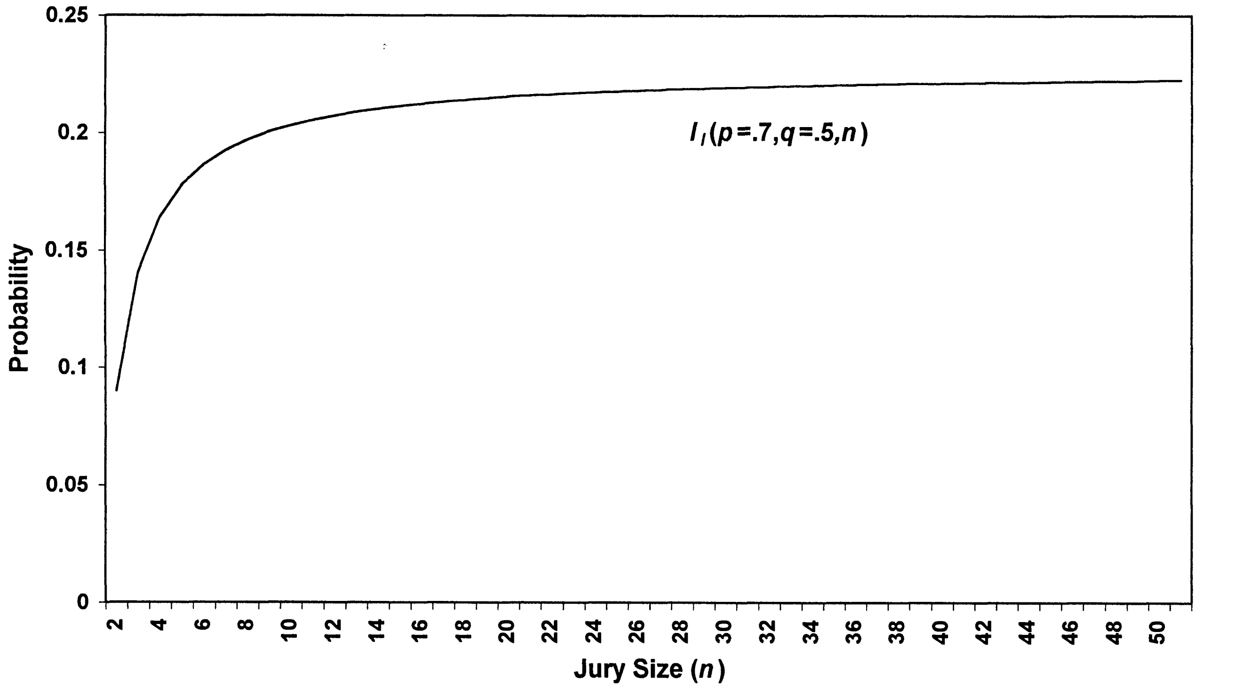
PROPOSITION 2. *Assume condition 1 holds and $q > 1 - p$. The strategy given by equation 3 is the unique responsive symmetric equilibrium for the unanimity rule. Moreover, $\sigma(i) \to 1$ as $n \to \infty$, and*

[9] Even when the unanimity rule is used the profile in which all jurors vote to acquit regardless of their signal is a symmetric equilibrium. Ruling out weakly dominated strategies does not eliminate this equilibrium.

[10] It is straightforward to verify that there are no mixed strategy equilibria in which jurors who receive signal g vote with positive probability to acquit. Support for such a mixed strategy equilibrium requires that those who receive a guilty signal are indifferent between convicting and acquitting, given their vote is pivotal. In such a case those who receive an innocent signal strictly prefer to acquit. But then the only jurors who vote to convict are those who receive the guilty signal. Under the unanimity rule, a vote is only pivotal when all others vote guilty. Thus, such a strategy profile is not a Nash equilibrium, by the same argument used to show that informative voting is not an equilibrium.

[11] Clearly, there also exist equilibria where the defendant is never convicted. For example, if all jurors vote to acquit independent of their signal, then we have a Nash equilibrium, since no juror can influence the outcome.

FIGURE 1. The Probability an Innocent Defendant Is Convicted as a Function of Jury Size



$$\lim_{n\to\infty} l_I(p, q, n) = \left(\frac{(1-q)(1-p)}{qp}\right)^{p/(2p-1)},$$

$$\lim_{n\to\infty} l_G(p, q, n) = 1 - \left(\frac{(1-q)(1-p)}{qp}\right)^{1-p/(2p-1)}.$$

If $q \leq 1 - p$, then there is no responsive equilibrium. In this case $\sigma(i) = 1$ is an equilibrium, and $l_I(p, q, n) = 1$, $l_G(p, q, n) = 0$.

Proof. For $q > 1 - p$ we demonstrated in the text above that the unique responsive voting equilibrium under the unanimity rule is given by equation 3. In Appendix A we show that

$$\lim_{n\to\infty} l_I(p, q, n)$$

$$= \lim_{n\to\infty}\left(\frac{(2p-1)\left(\frac{(1-q)(1-p)}{qp}\right)^{1/(n-1)}}{p-(1-p)\left(\frac{(1-q)(1-p)}{qp}\right)^{1/(n-1)}}\right)^{n}$$

$$= \left(\frac{(1-q)(1-p)}{qp}\right)^{p/(2p-1)}.$$

The proof that $\lim_{n\to\infty} l_G(p, q, n) = 1 - ((1 - q)(1 - p)/qp)^{(1-p)/(2p-1)}$ is analogous.

If $q \geq 1 - p$, then the argument given in the text shows that there is no responsive equilibrium. *Q.E.D.*

Proposition 2 also implies that the probability of a guilty defendant being convicted $(1 - l_G)$ is bounded away from zero for all n. This is again in contrast to the case of informative voting, in which the probability of conviction converges to zero as $n \to 0$, independent of whether the defendant is guilty or innocent. Thus, a second implication of strategic voting is that the probability of a guilty verdict may be much larger than under informative voting.

To provide an intuition for proposition 2, first observe that equation 3 implies $\sigma(i) \to 1$ as $n \to \infty$. As a consequence, γ_G (the probability that a juror votes to convict if the defendant is guilty) and γ_I (the probability that a juror votes to convict if the defendant is innocent) both converge to one. This is not enough to show that the probability of convicting an innocent defendant, $(\gamma_I)^n$, stays bounded away from zero. In Appendix A we demonstrate that for large n, γ_I can be approximated by

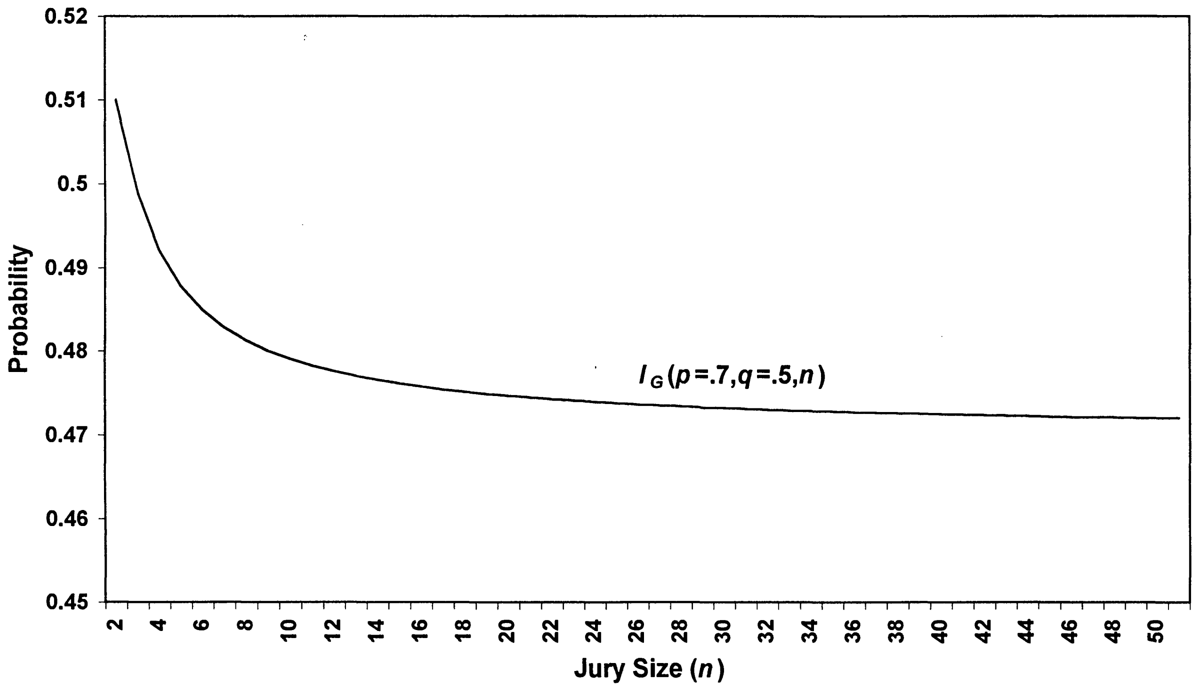
$$1 + \frac{1}{n-1}\left(\frac{p}{2p-1}\ln f\right),$$

where $f = (1 - q)(1 - p)/qp$, and hence $(\gamma_I)^n$ converges to $f^{p/(2p-1)}$, which is the bound given in proposition 2.[12]

The convergence to the bounds given in proposition 2 is fast, and hence the limit formula allows us to approximate the probabilities of each kind of error even for small juries. Figure 1 illustrates the convergence of $l_I(p, q, n)$ for the values $p = 0.7$, $q = 0.5$. The figure is startling for several reasons. First, the limit probability of convicting an innocent defendant is quite large—22%. Second, when there are only 12 jurors the probability of convicting an innocent is 21%.

[12] Recall that $e^{x'} = \lim_{n\to\infty}(1 + x/n)^n$.

FIGURE 2. The Probability a Guilty Defendant Is Acquitted as a Function of Jury Size



There is a difference of only 1% between the probability of convicting an innocent defendant with a jury size of $n \geq 12$ and the limit probability. Third, the probability of convicting an innocent defendant *increases* with the size of the jury.

Figure 2 shows the probability of acquitting a guilty defendant for the values $p = 0.7$, $q = 0.5$. The limit probability of acquitting a guilty defendant is 47%. Once again, the limit probability is a very good estimate of the actual probability of this type of mistake, even for small juries. Note that the probability of acquitting a guilty defendant actually *decreases* with the size of the jury.

Figure 3 shows the limit errors $l_I(p, q) = \lim_{n\to\infty} l_I(p, q, n)$ and $l_G(p, q) = \lim_{n\to\infty} l_G(p, q, n)$ for the value $p = 0.7$ as a function of q. The figure demonstrates that for large juries the probability of convicting an innocent decreases in q, while the probability of acquitting a guilty defendant increases in q. Thus, the unanimity rule does a poor job of protecting innocent defendants from unreasonable juries. Yet, if the jury is responsible (e.g., $q = .9$), the innocent defendant is protected at the cost of a high probability of acquitting the guilty. In the next section we show that the unanimity rule is a uniquely bad voting rule in terms of the probabilities of both kinds of error it induces.

Using the equilibrium strategies calculated in this section, we may also compute the probability that a convicted defendant is innocent and compare it to the bound given in proposition 1. A straightforward calculation demonstrates that

$$\lim_{n\to\infty} \Pr(G|C) = \frac{1-q}{1+q\,\dfrac{2p-1}{1-p}} > \frac{1-q}{1+q\,\dfrac{2p-1}{(1-p)^2}},$$

where the right-hand side of the above inequality is the bound given in proposition 1.

NONUNANIMOUS RULES

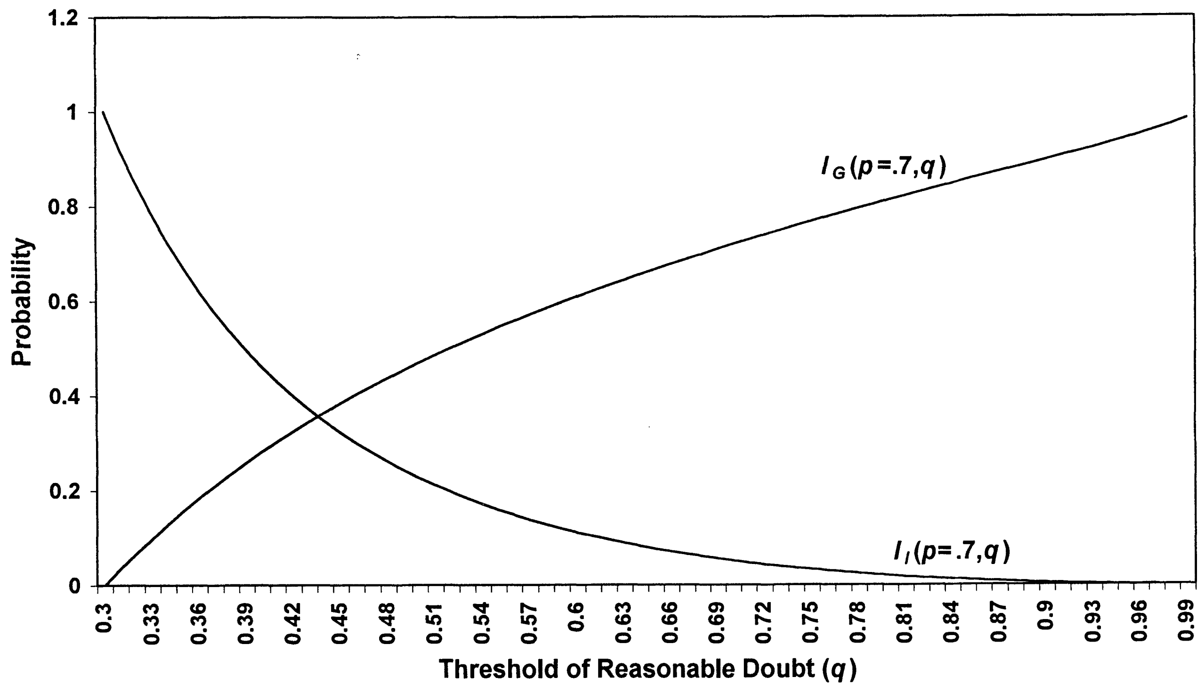
We now analyze the probability of making the wrong decision under nonunanimous voting rules.[13] Suppose $\hat{k} = \alpha n$ for some α with $0 < \alpha < 1$, and assume that αn is an integer. Thus, a defendant will be convicted if and only if at least an α-fraction of the jury votes to convict. For a fixed α, consider a sequence of symmetric responsive equilibria corresponding to an increasing jury size. In the following proposition we show that as $n \to \infty$ the probability of making either of the two kinds of error converges to zero.

PROPOSITION 3. *Fix any α with $0 < \alpha < 1$. (1) There is a n' such that for $n > n'$ there is a symmetric responsive equilibrium. (2) For any sequence of symmetric responsive equilibria the probability of convicting an innocent defendant and the probability of acquitting a guilty defendant both converge to zero.*

Proof. See Appendix B. *Q.E.D.*

[13] See Appendix B for a computation of the (unique) symmetric responsive equilibrium for general $\hat{k}$.

FIGURE 3. Limit Error Probabilities as a Function of the Threshold of Reasonable Doubt



Proposition 3 shows that for any $\alpha \in (0, 1)$ the probability of convicting an innocent defendant and the probability of acquitting a guilty defendant both converge to zero as the jury grows large.[14] This is in sharp contrast to the results of propositions 1 and 2, which showed that both types of mistakes stay bounded away from zero for the unanimity rule.

To provide an intuition for proposition 3, recall that γ_G is the probability a juror votes to convict if the defendant is guilty and γ_I is the corresponding probability if the defendant is innocent. As we noted above, in any responsive profile it must be the case that $1 > \gamma_G > \gamma_I > 0$. Suppose the actual fraction of guilty votes is a. Then probability that the defendant is guilty is given by

$$\frac{(\gamma_G)^{an}(1-\gamma_G)^{(1-a)n}}{(\gamma_G)^{an}(1-\gamma_G)^{(1-a)n} + (\gamma_I)^{an}(1-\gamma_I)^{(1-a)n}}.$$

It follows that if

$$\frac{(\gamma_I)^{a}(1-\gamma_I)^{1-a}}{(\gamma_G)^{a}(1-\gamma_G)^{1-a}} \neq 1,$$

then for large n the defendant is either guilty with probability close to one (if the above fraction is less than one) or innocent with probability close to one (if the above fraction is greater than one).

For any responsive profile there is a unique a^*, such that

$$\frac{(\gamma_I)^{a^*}(1-\gamma_I)^{1-a^*}}{(\gamma_G)^{a^*}(1-\gamma_G)^{1-a^*}} = 1,$$

and $\gamma_G > a^* > \gamma_I$.[15] If the actual fraction of guilty votes is $a < a^*$, then for large n the defendant is innocent with probability close to one; if $a > a^*$, then the defendant is guilty with probability close to one.

In any responsive equilibrium it must be the case that the event a vote is pivotal, that is, an α fraction votes to convict, is not overwhelming evidence of either guilt or innocence.[16] This in turn implies that the a^* implied by a responsive equilibrium must be arbitrarily close to α if n is sufficiently large.

Now recall that a responsive equilibrium may take two possible forms. One is that jurors vote to acquit when they observe signal i and randomize when they observe signal g (i.e., $\sigma(i) = 0$ and $0 \leq \sigma(g) \leq 1$). The other is that jurors vote to convict when they observe signal g and randomize when they observe signal i (i.e., $0 \leq \sigma(i) \leq 1$ and $\sigma(g) = 1$). Figure 4 depicts γ_I, γ_G, and a^* as a function of the strategy profile for $p = 0.7$. It is convenient to represent the strategy by the variable $x \in [0, 2]$. For $x \leq 1$ the strategy is $\sigma(i) = 0$, $\sigma(g) = x$; for $x \geq 1$ the strategy is $\sigma(g) = 1$, $\sigma(i) = x - 1$.

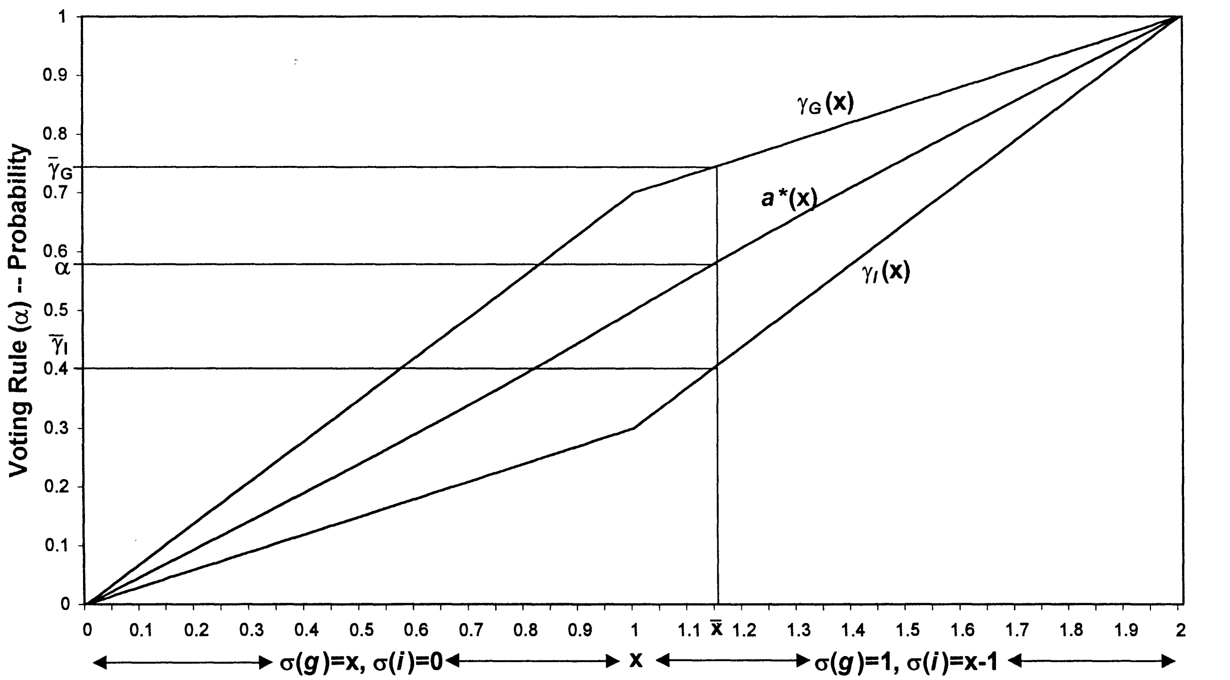
Figure 4 allows us to find the unique symmetric responsive equilibrium for large juries as a function of the voting rule α. Suppose, for example, that α is as indicated in the figure. Since a^* in a large jury is close to α, it must be that the equilibrium strategy profile is

[14] Proposition 3 holds in much more general environments. Feddersen and Pesendorfer (1997a) prove the analogous result for an environment that includes preference diversity and a much broader range of information environments. Myerson (n.d.) proves a similar result for the case of simple majority rule.

[15] It is easy to see that $(\gamma_I)^{a^*}(1-\gamma_I)^{1-a^*}/(\gamma_G)^{a^*}(1-\gamma_G)^{1-a^*} = 1$ implies $a^* = (\ln(1-\gamma_G) - \ln(1-\gamma_I))/(\ln\gamma_I - \ln\gamma_G + \ln(1-\gamma_G) - \ln(1-\gamma_I))$.

[16] Formally, it must be the case in any responsive equilibrium that $\Pr(G|piv, g) \geq q \geq \Pr(G|piv, i)$.

FIGURE 4. Limit Equilibrium Strategies as a Function of the Voting Rule



Note: Calculations assume $p = .7$ and $0 < q < 1$.
The actual functions shown are:

$$\gamma_G(x)=\begin{cases} .7+.3(x-1) & \text{if } x\geq 1 \\ .7x & \text{if } x<1 \end{cases}$$

$$\gamma_I(x)=\begin{cases} .3+.7(x-1) & \text{if } x\geq 1 \\ .3x & \text{if } x<1 \end{cases}$$

$$a^*(x)=\frac{\ln(1-\gamma_G(x))-\ln(1-\gamma_I(x))}{\ln \gamma_I(x)-\ln \gamma_G(x)+\ln(1-\gamma_G(x))-\ln(1-\gamma_I(x))}$$

close to $\bar{x}$ (which corresponds to $\sigma(g) = 1$ and $\sigma(i) = \bar{x} - 1$). Figure 4 also illustrates why large juries rarely make mistakes when $0 < \alpha < 1$. Observe that for large n the equilibrium pair (γ_G, γ_I) is very close to $(\bar{\gamma}_G, \bar{\gamma}_I)$, and $\bar{\gamma}_G > \alpha > \bar{\gamma}_I$. Furthermore, as the size of the jury grows the actual vote share converges in probability to the expected vote share. Therefore, when the defendant is guilty, the actual fraction of guilty votes will be close to $\bar{\gamma}_G$, with probability close to one, and the defendant will be convicted. Conversely, when the defendant is innocent, the actual fraction of guilty votes will be close to $\bar{\gamma}_I$, with probability close to one, and the defendant will be acquitted. By contrast, consider the unanimity rule ($\alpha = 1$). In order for a^* to be close to one it is necessary that both γ_G and γ_I be close to one, and therefore the above argument fails.

Propositions 1, 2, and 3 imply that the unanimity rule is *uniquely bad* for large juries. A second conclusion that can be drawn from proposition 3 is that under nonunanimous rules the size of a jury is more important in determining the probability of making a mistake in the verdict than is the voting rule. Therefore, if the probability an innocent defendant is convicted is considered to be too large under a nonunanimous rule, then the remedy is not to change the rule but to increase the size of the jury.

In the next section we provide an example which suggests that convergence is fast, and hence our convergence results are indeed relevant for relatively small juries.

EXAMPLE

We consider a 12-person jury, $n = 12$. We set the parameter for reasonable doubt at $q = 0.9$, that is, jurors need to believe that the defendant is guilty with probability 0.9 in order to convict. We assume that $p = 0.8$, that is, the probability of receiving a guilty signal if the defendant is guilty is 0.8.[17]

The probability of convicting an innocent defendant, $l_I(\hat{k})$, is given by

$$l_I(\hat{k}) = \sum_{j=\hat{k}}^{n} \binom{n}{j} (\gamma_I(\hat{k}))^j (1 - \gamma_I(k))^{n-j},$$

where

$$\gamma_I(\hat{k}) = (1-p)\sigma(g)(\hat{k}) + p\sigma(i)(\hat{k})$$

is the probability that any juror votes for conviction if the defendant is innocent.

[17] It follows that a jury of one would never convict, since $\beta(1, 1) = .8 < .9 = q$.

TABLE 1. An Example of Error Probabilities and Equilibrium Strategies in a 12-Person Jury When $\hat{k} \geq 7$

Voting Rule $\hat{k}$	7	8	9	10	11	12
l_I	.004	.0011	.0025	.0045	.0066	.0069
l_G	.019	.0660	.1350	.2450	.4200	.6540
$\sigma(i)$	0	.0230	.1430	.2770	.4230	.5750

Note: We assume that $p = .8$ and $q = .9$. Note that l_I is the probability an innocent defendant is convicted and l_G is the probability a guilty defendant is acquitted. The full equilibrium profile is $\sigma(g) = 1$ and $\sigma(i)$ as specified in the table.

Similarly, the probability of acquitting a guilty defendant, $l_G(\hat{k})$, is given by

$$l_I(\hat{k}) = 1 - \sum_{j=\hat{k}}^{n} \binom{n}{j} (\gamma_G(\hat{k}))^j (1 - \gamma_G(k))^{n-j},$$

where

$$\gamma_G(\hat{k}) = p\sigma(g)(\hat{k}) + (1 - p)\sigma(i)(\hat{k}).$$

Table 1 gives the probability of making mistakes as a function of the rule $\hat{k}$ when $\hat{k} \geq 7$. At $\hat{k} = 7$, informative voting is the equilibrium. Thus, every juror who receives a signal g votes to convict, and every juror who receives a signal i votes to acquit. For all $\hat{k} > 7$, the jurors who receive the signal i mix between voting to convict and voting to acquit, while those who receive signal g always vote to convict. As the table shows, the unanimity rule has the highest probability of convicting an innocent defendant when $\hat{k} \geq 7$. In addition, all rules with $\hat{k} \geq 7$ have the property that they lead to a lower probability of acquitting a guilty defendant than the unanimity rule.

Table 2 shows the probability of making an error when $\hat{k} < 7$. For $\hat{k} \leq 6$ the equilibrium strategies are such that a juror who receives the signal i always votes to acquit, while a juror who receives the signal g mixes between voting to convict and voting to acquit. For $\hat{k} = 1$ the defendant is never convicted, that is, $l_G = 1$. To see why this is the case, suppose that no juror ever votes to convict. In this case, each juror is always pivotal, since one guilty vote is enough for a conviction. But this implies that the only information the juror has conditional on his vote being pivotal is his own signal. Conditional on having received signal g, the juror believes that the defendant is guilty with probability 0.8. Since the reasonable doubt threshold is 0.9, each juror votes to acquit.

TABLE 2. Error Probabilities and Equilibrium Strategies in a 12-Person Jury When $\hat{k} < 7$

Voting Rule	1	2	3	4	5	6
l_I	0	.0159	.034	.027	.019	.010
l_G	1	.780	.410	.210	.095	.036
$\sigma(g)$	0	.091	.306	.512	.704	.879

Note: The full equilibrium profile is $\sigma(i) = 0$ and $\sigma(g)$ as specified in the table.

The probability of convicting an innocent defendant reaches a maximum at $\hat{k} = 3$, while the probability of acquitting a guilty defendant is monotonically decreasing for $\hat{k} \leq 6$, and the probability of convicting an innocent defendant is zero at $\hat{k} = 1$.[18]

CONCLUSION

We demonstrated that strategic behavior dramatically alters our intuitions about the consequences of jury voting rules. When jurors vote strategically, the unanimity rule results in a strictly positive probability both of acquitting the guilty and convicting the innocent, even for very large juries. Increasing the size of the jury does not help and actually may increase the probability of convicting an innocent defendant. Finally, in large juries the unanimity rule is inferior to a variety of other rules.

It is appropriate to conclude with a note of caution. Jury reform is not an abstract proposition. A group called Citizens for a Safer California proposed the Public Safety Protection Act of 1996. It would eliminate the unanimous verdict in all but capital murder cases and replace it with a rule requiring only 10 of 12 jurors to convict.[19] Our results lend some support to such an initiative. Yet, retaining the unanimity rule in capital cases is exactly the wrong thing to do. Presumably, the motive for retaining it is to protect against the terrible consequences of convicting an innocent. Our results suggest that it would be better to combine a supermajority rule with a larger jury for cases in which it is desirable to reduce the probability of convicting an innocent.

More important, our results depend on the assumptions of private information and strategic voting. As noted above, there is some experimental evidence that strategic voting will occur in the presence of private information and common values (Ladha, Miller, and Oppenheimer 1996). There is also some empirical support for the assumption that deliberations fail to eliminate private information, and hence jurors have private information at the final voting stage. Selvin and Picus (1987, 24) conducted interviews with jurors after the verdict and found significant differences in their information about the facts of the trial. The degree to which strategic voting and private information characterize actual juries is ultimately an empirical question and beyond the scope of this article.

The final caveat is that criminal trials in the United States have at least three possible outcomes: convic-

[18] In this case, conditional on a vote being pivotal, a juror knows that all other jurors must have voted to acquit. In equilibrium this can never be information in favor of the guilt of the defendant. If the juror receives a guilty signal, he therefore believes the defendant to be guilty with probability at most p. Since $p < q$, the juror votes to acquit.

[19] The group claims that a "broad coalition of crime victims, law enforcement and concerned citizens" support the proposed legislation. It also claims that an independent poll shows 71% of Californians support 10:2 jury verdicts. This information was downloaded from the Internet at http://taren.ns.net/cdaa.htm, October 1996.

tion, acquittal, or mistrial. A unanimous vote is required for either conviction or acquittal. The effect of strategic voting and private information in this setting has been examined by Coughlan (1997). He shows that requiring juries to reach a unanimous verdict minimizes the probability of both types of error when a hung jury always results in a retrial. As we showed above, however, the probability of errors can be made arbitrarily small when a single jury verdict is decisive by using a nonunanimous rule and increasing the size of the jury. Hence, if retrying the defendant is very costly, unanimity remains an inferior rule.

APPENDIX A

Proof of Proposition 1

Let $\sigma = \{(\sigma_j(i), \sigma_j(g)\}_{j=1}^n$ be a Nash equilibrium profile. There are two types of equilibria in which the defendant is convicted with positive probability. The first type occurs when $1 - p > q$ and all jurors vote to convict independent of their private signal. In this case for any juror j it follows that $\Pr(G|piv_j, i) = 1 - p \geq q$. The defendant is always convicted, and the probability that a convicted defendant is innocent is equal to 1/2.

The second type of equilibrium occurs when there is some juror who votes to acquit with positive probability. Suppose j upon receiving signal s votes to acquit with positive probability, that is, $\sigma_j(s) < 1$. Also note that since the defendant is convicted with strictly positive probability, it follows that each juror is pivotal with strictly positive probability. Since $\sigma_j(s) < 1$, it follows that

$$
\begin{aligned}
q &\geq \Pr(G|piv_j, s) \\
&= \frac{\Pr(piv_j|G)\Pr(s|G)(1/2)}{\Pr(piv_j|G)\Pr(s|G)(1/2) + \Pr(piv_j|I)\Pr(s|I)(1/2)} \\
&= \frac{\dfrac{\Pr(piv_j|G)}{\Pr(piv_j|G) + \Pr(piv_j|I)}\Pr(s|G)}{\left[\dfrac{\Pr(piv_j|G)}{\Pr(piv_j|G) + \Pr(piv_j|I)}\Pr(s|G) + \dfrac{\Pr(piv_j|I)}{\Pr(piv_j|G) + \Pr(piv_j|I)}\Pr(s|I)\right]} \\
&= \frac{\Pr(G|piv_j)\Pr(s|G)}{\Pr(G|piv_j)\Pr(s|G) + (1 - \Pr(G|piv_j))\Pr(s|I)}.
\end{aligned}
$$

Thus,

$$\Pr(G|piv_j) \leq \frac{qp}{(1-q)(1-p) + qp}. \tag{4}$$

Recall that C denotes the event that the defendant is convicted, and let $\gamma_{Gj} = p\sigma_j(g) + (1 - p)\sigma_j(i)$ and $\gamma_{I_j} = p\sigma_j(i) + (1 - p)\sigma_j(g)$. Since the defendant is convicted with strictly positive probability, it follows that $\sigma_j(g) > 0$ or $\sigma_j(i) > 0$; therefore, both $\gamma_{Gj} > 0$ and $\gamma_{Ij} > 0$. Hence we may write

$$\Pr(G|C) = \frac{\Pr(G|piv_j)\gamma_{Gj}}{\Pr(G|piv_j)\gamma_{Gj} + (1 - \Pr(G|piv_j))\gamma_{Ij}}.$$

Since

$$\frac{\gamma_{Ij}}{\gamma_{Gj}} \geq \frac{1-p}{p},$$

it follows from 4 and some algebra that

$$\Pr(G|C) \leq \frac{qp^2}{1 - 2p + p^2 - q + 2qp},$$

which gives the result

$$\Pr(I|C) = 1 - \Pr(G|C) \geq \frac{(1-p)^2(1-q)}{1 - 2p + p^2 - q + 2qp}.$$

Q.E.D.

Proof of Proposition 2

We now show that

$$\lim_{n\to\infty}\left(\frac{1}{\dfrac{p}{2p-1}\left(\dfrac{(1-q)(1-p)}{qp}\right)^{-1/n-1} - \dfrac{(1-p)}{2p-1}}\right)^n = \left(\frac{(1-q)(1-p)}{qp}\right)^{p/(2p-1)}.$$

Let

$$h = \frac{p}{2p-1}\left(\frac{(1-q)(1-p)}{qp}\right)^{-1/(n-1)} - \frac{(1-p)}{2p-1}.$$

Suppose $\infty > \lim_{n\to\infty} h^n > 0$, then $\lim_{n\to\infty} h^{-n} = 1/\lim_{n\to\infty} h^n$. Now let

$$f = \frac{(1-q)(1-p)}{qp}.$$

We use the following facts:

$$\lim_{n\to\infty}\left(1 + \frac{z}{n}\right)^n = e^z, \tag{5}$$

and, given $f \in (0, 1)$,

$$1 + \frac{-1}{n}\ln f \geq f^{-1/n-1} \geq 1 + \frac{-1}{n-1}\ln f. \tag{6}$$

From 6 we know

$$h \geq \frac{p}{2p-1}\left(1 + \frac{-1}{n-1}\ln f\right) - \frac{(1-p)}{2p-1}.$$

Some simple algebra shows that

$$\frac{p}{2p-1}\left(1 + \frac{-1}{n-1}\ln f\right) - \frac{(1-p)}{2p-1} = 1 + \frac{1}{n-1}\frac{-p}{2p-1}\ln f.$$

Thus,

$$\lim_{n\to\infty}(h)^n \geq \lim_{n\to\infty}\left(1 + \frac{1}{n-1}\frac{-p}{2p-1}\ln f\right)^n,$$

and from 5 we get

$$\lim_{n\to\infty}\left(1 + \frac{1}{n-1}\frac{-p}{2p-1}\ln f\right)^n = \left(\frac{(1-q)(1-p)}{qp}\right)^{-p/(2p-1)}.$$

We can make an identical argument using 6 to establish that

$$\lim_{n\to\infty} (h)^n \le \lim_{n\to\infty} \left(1 + \frac{1}{n}\frac{-p}{2p-1}\ln f\right)^n$$

$$= \left(\frac{(1-q)(1-p)}{qp}\right)^{-p/(2p-1)}. \qquad Q.E.D.$$

APPENDIX B

First, we compute the equilibrium for a general $\hat{k}$. Denote the probability that a juror votes to convict in state I as

$$\gamma_I = (1-p)\sigma(g) + p\sigma(i)$$

and the probability that a juror votes to convict in state G as

$$\gamma_G = p\sigma(g) + (1-p)\sigma(i).$$

When $1 > \sigma(i)$, we must have

$$\frac{1}{1 + \dfrac{p(\gamma_I)^{\hat{k}-1}(1-\gamma_I)^{n-\hat{k}}}{(1-p)(\gamma_G)^{\hat{k}-1}(1-\gamma_G)^{n-\hat{k}}}} \le q, \tag{7}$$

with equality holding if $1 > \sigma(i) > 0$. Similarly, when $\sigma(g) > 0$ it must be true that

$$\frac{1}{1 + \dfrac{(1-p)(\gamma_I)^{\hat{k}-1}(1-\gamma_I)^{n-\hat{k}}}{p(\gamma_G)^{\hat{k}-1}(1-\gamma_G)^{n-\hat{k}}}} \ge q, \tag{8}$$

with equality holding if $1 > \sigma(g) > 0$.

Second, we show that $1 > \sigma(g) > 0$ implies $\sigma(i) = 0$ (an identical exercise shows that $1 > \sigma(i) > 0$ implies $\sigma(g) = 1$). Suppose $1 > \sigma(g) > 0$; 8 then implies

$$\frac{(1-q)p}{q(1-p)} = \frac{(\gamma_I)^{\hat{k}-1}(1-\gamma_I)^{n-\hat{k}}}{(\gamma_G)^{\hat{k}-1}(1-\gamma_G)^{n-\hat{k}}}.$$

Since $p > 1/2$ we can rewrite 7 as

$$\frac{1}{1 + \dfrac{(1-q)}{q}\left(\dfrac{p}{1-p}\right)^2} = q\frac{p^2 - (2p-1)}{p^2 - q(2p-1)} < q,$$

which implies $\sigma(i) = 0$.

Thus, in any responsive equilibrium we must have either $\sigma(i) = 0$ and $\sigma(g) > 0$ or $\sigma(i) < 1$ and $\sigma(g) = 1$. If

$$\frac{(1-p)(p)^{\hat{k}-1}(1-p)^{n-\hat{k}}}{p(1-p)^{\hat{k}-1}p^{n-\hat{k}} + (1-p)p^{\hat{k}-1}(1-p)^{n-\hat{k}}}$$

$$= \frac{1}{1 + \dfrac{p(p)^{\hat{k}-1}(1-p)^{n-\hat{k}}}{(1-p)(p)^{\hat{k}-1}(1-p)^{n-\hat{k}}}} \le q \tag{9}$$

and

$$\frac{(p)^{\hat{k}}(1-p)^{n-\hat{k}}}{(1-p)^{\hat{k}}p^{n-\hat{k}} + p^{\hat{k}}(1-p)^{n-\hat{k}}}$$

$$= \frac{1}{1 + \dfrac{(1-p)(p)^{\hat{k}-1}(1-p)^{n-\hat{k}}}{p(p)^{\hat{k}-1}(1-p)^{n-\hat{k}}}} \ge q, \tag{10}$$

then the unique responsive voting equilibrium is $\sigma(i) = 0$ and $\sigma(g) = 1$. (Recall that a voting equilibrium is a symmetric Nash equilibrium.)

To see why this is the unique responsive voting equilibrium, observe that the left-hand side of 7 is strictly decreasing in $\sigma(i)$. Together with 9 this implies that whenever $\sigma(i) > 0$ (and $\sigma(g) = 1$) every juror has a strict incentive to vote to acquit. Similarly, the left-hand side of 8 is strictly decreasing in $\sigma(g)$. This together with 10 implies that whenever $\sigma(g) < 1$ (and $\sigma(i) = 0$) every juror has a strict incentive to vote to convict.

If either 9 or 10 does not hold, then there are two cases to consider.

Case 1

Suppose

$$\frac{(p)^{\hat{k}}(1-p)^{n-\hat{k}}}{(1-p)^{\hat{k}}p^{n-\hat{k}} + p^{\hat{k}}(1-p)^{n-\hat{k}}} < q.$$

Then $\sigma(i) = 0$ is the equilibrium, and the equilibrium condition for $\sigma(g)$ is defined by 8 with equality holding. This yields

$$\frac{1}{1 + \dfrac{(1-p)(\gamma_I)^{k-1}(1-\gamma_I)^{n-k}}{p(\gamma_G)^{k-1}(1-\gamma_G)^{n-k}}} = q,$$

which we can rewrite as

$$\frac{(1-q)p}{q(1-p)} = \frac{(\gamma_I)^{k-1}(1-\gamma_I)^{n-k}}{(\gamma_G)^{k-1}(1-\gamma_G)^{n-k}}$$

$$= \frac{((1-p)\sigma)^{k_n-1}(1-(1-p)\sigma)^{n-k_n}}{(p\sigma)^{k_n-1}(1-p\sigma))^{n-k_n}}$$

$$= \left(\frac{1-p}{p}\right)^{k-1}\left(\frac{1-(1-p)\sigma}{1-p\sigma}\right)^{n-k}.$$

Therefore, we get

$$\frac{(1-q)}{q}\left(\frac{p}{1-p}\right)^k = \left(\frac{1-(1-p)\sigma}{1-p\sigma}\right)^{n-k}$$

$$\frac{(1-\sigma+p\sigma)}{(1-p\sigma)} = \left(\frac{(1-q)}{q}\left(\frac{p}{1-p}\right)^k\right)^{1/(n-k)}.$$

This yields

$$\sigma(g) = \frac{h-1}{p(h+1)-1},$$

where

$$h = \left(\frac{(1-q)}{q}\left(\frac{p}{1-p}\right)^k\right)^{1/(n-k)}.$$

Clearly, since $\sigma(g)$ is the unique solution of 8 in this case, there is a unique responsive voting equilibrium in this case.

Case 2

Suppose

$$\frac{(1-p)(p)^{\hat{k}-1}(1-p)^{n-\hat{k}}}{p(1-p)^{\hat{k}-1}p^{n-\hat{k}} + (1-p)p^{\hat{k}-1}(1-p)^{n-\hat{k}}} > q.$$

In this case $\sigma(g) = 1$, and the equilibrium condition is given by:

$$\frac{(1-p)(\gamma_G)^{\hat{k}-1}(1-\gamma_G)^{n-\hat{k}}}{p(\gamma_I)^{\hat{k}-1}(1-\gamma_I)^{n-\hat{k}} + (1-p)(\gamma_G)^{\hat{k}-1}(1-\gamma_G)^{n-\hat{k}}} \le q,$$

with equality holding whenever $\sigma(i) \in (0, 1)$.

A straightforward calculation shows that for an interior solution in this case

$$\sigma(i) = \frac{p(1+f)-1}{p-f(1-p)},$$

where

$$f = \left(\frac{(1-q)}{q}\left(\frac{(1-p)}{p}\right)^{n-k+1}\right)^{1/(k-1)}.$$

Again, since $\sigma(i)$ is the unique solution of 7 in this case, there is a unique responsive voting equilibrium in this case.

Whenever $\sigma(i)$ as defined by the previous two equations is less than zero, then $\sigma(i) = 0$; whenever $\sigma(i)$ as defined by the previous two equations is larger than 1, then $\sigma(i) = 1$.

Proof of Proposition 2

For sufficiently large n there exists a responsive voting equilibrium. To see this we first compute the limit equilibrium as $n \to \infty$ for the case in which $k/n = \alpha$. In case 2 we have

$$\sigma(i) = \frac{p(1+f)-1}{p-f(1-p)},$$

where

$$f = \lim_{n\to\infty}\left(\frac{(1-q)}{q}\left(\frac{(1-p)}{p}\right)^{n-\alpha n+1}\right)^{1/(\alpha n-1)} = \left(\frac{1-p}{p}\right)^{1-\alpha/\alpha},$$

and therefore we have

$$\sigma(i) = \frac{p(1+f)-1}{p-f(1-p)} = \frac{p\left(1+\left(\frac{1-p}{p}\right)^{(1-\alpha)/\alpha}\right)-1}{p-\left(\frac{1-p}{p}\right)^{(1-\alpha)/\alpha}(1-p)}.$$

It is easily checked that $1 > \sigma(i) \geq 0$ for $1 > \alpha \geq 1/2$ with $\sigma(i) \to 1$ as $\alpha \to 1$. Similarly, in case 1 we have

$$\sigma(g) = \frac{\left(\frac{p}{1-p}\right)^{\alpha/(1-\alpha)}-1}{p\left(\left(\frac{p}{1-p}\right)^{\alpha/(1-\alpha)}+1\right)-1},$$

and again it can easily be checked that $0 < \sigma(g) \leq 1$ for $0 < \alpha \leq 1/2$ with $\sigma(g) \to 0$ as $\alpha \to 0$.

Together this implies that for any $0 < \alpha < 1$ there is a responsive limit equilibrium. Now a simple continuity argument implies that for sufficiently large n the solution to equations 7 and 8 must be arbitrarily close to the limit solution, and hence it follows that for sufficiently large n there is a responsive voting equilibrium.

To prove part (2) of proposition 3, observe that in the limit as $n \to \infty$ we have

$$\gamma_I = 1-p+p\,\frac{p\left(1+\left(\frac{1-p}{p}\right)^{(1-\alpha)/\alpha}\right)-1}{p-\left(\frac{1-p}{p}\right)^{(1-\alpha)/\alpha}(1-p)}$$

and

$$\gamma_G = p+(1-p)\,\frac{p\left(1+\left(\frac{1-p}{p}\right)^{(1-\alpha)/\alpha}\right)-1}{p-\left(\frac{1-p}{p}\right)^{(1-\alpha)/\alpha}(1-p)}.$$

Note that for $0 < \alpha < 1$ this implies that

$$\gamma_G > \gamma_I - \varepsilon,$$

for some $\varepsilon > 0$ which depends on α.

Next we show that

$$\gamma_I < \alpha < \gamma_G. \tag{11}$$

To see why this is sufficient to prove proposition 3, note that by the law of large numbers the actual share of guilty votes converges to the expected share of guilty votes in each state. Hence, the share of guilty votes if the defendant is innocent converges to $\gamma_I < \alpha$ in probability, and the defendant is acquitted with probability close to one for large n. Similarly, if the defendant is guilty, the share of guilty votes converges to $\gamma_G > \alpha$ in probability, and hence the defendant is convicted with probability close to one for large n.

Suppose 11 is violated, and $\gamma_G > \gamma_I \geq \alpha$. From the equilibrium conditions we know that for all n

$$\frac{1}{1+\dfrac{(1-p)(g_{ni})^{\alpha n-1}(1-g_{ni})^{(1-\alpha)n}}{p(g^n_{ng})^{\alpha n-1}(1-g^n_{ng})^{(1-\alpha)n}}} \geq q \tag{12}$$

(where g_{ns} denotes the equilibrium probability of a guilty vote in state s with n jurors). Note that $x^\alpha(1-x)^{(1-\alpha)}$ is a single peaked function for $x \in [0, 1]$ with a maximum at $x = \alpha$. If $\gamma_G > \gamma_I \geq \alpha$, then $(\gamma_I)^\alpha(1-\gamma_I)^{(1-\alpha)}/(\gamma_G)^\alpha(1-\gamma_G)^{(1-\alpha)} > 1$, and hence

$$\left(\frac{(\gamma_I)^\alpha(1-\gamma_I)^{(1-\alpha)}}{(\gamma_G)^\alpha(1-\gamma_G)^{(1-\alpha)}}\right)^n \to \infty.$$

The left-hand side of 12 must converge to zero as $n \to \infty$, and hence inequality 12 cannot hold. An analogous argument can be made if $\alpha \geq \gamma_G > \gamma_I$. *Q.E.D.*

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