Abstract:

Given N equations in M unknowns, a helpful intuition is that the solution space should have dimension M-N. Unfortunately, many useful and interesting spaces break this model -- they require many, many equations, so that even basic geometric facts (like their size) are unclear. Hilbert, in the 1890s, found a solution: he looked at partial redundancies in systems of equations -- called "syzygies" -- and introduced the Hilbert polynomial, an invariant that accurately computes the dimension (and more) of the space of solutions.

We'll talk about Hilbert polynomials and their refinements: Betti tables, which detect a wealth of geometric and algebraic data. My own research is on classifying Betti tables -- outlining the behaviors we can encounter -- for matrix varieties, determinantal loci and other spaces coming from linear algebra.