Abstract:

Elliptic curves are solution sets to nonsingular cubic equations in two variables. Questions of how to find, count, and characterize points on elliptic curves have been studied since the days of Diophantus. Nearly 2000 years later, the great mathematician Joe Silverman wrote, “the theory of elliptic curves is rich, varied, and amazingly vast.” In this talk, I will briefly explore elliptic curves over three fields: the field of rational numbers, finite fields, and the field of complex numbers. Over the rationals, I will discuss how to generate new points from known ones. Over finite fields, I will define what it means to be a point on an elliptic curve and answer the question “How many?” Finally, I will explain how elliptic curves over the complex numbers are actually doughnuts.