

BI-CO MATHEMATICS COLLOQUIUM

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“Around the Moebius Function”

Monday, March 23, 2015

Talk at 4:00 – Park 338
Tea at 3:30 – Park 355, Math Lounge

Abstract:

The Moebius function plays a central role in number theory; both the prime number theorem and the Riemann Hypothesis are naturally formulated in terms of the amount of cancellations one gets when summing the Moebius function. In recent joint work with K. Matomaki we have shown that the sum of the Moebius function exhibits cancellations in “almost all intervals” of increasing length. This goes beyond what was previously known even conditionally on the Riemann Hypothesis and allows us to settle a long-standing conjecture on correlations of consecutive values of the Moebius function. Our result holds in fact in greater generality. Exploiting this generality we show that between a fixed number of consecutive squares there is always an integer composed of only “small” prime factors. This settles an old problem on “smooth numbers” and is related to the running time of Lenstra's factoring algorithm. Finally, in recent on-going work with K. Matomaki and T. Tao we have been able to use the previously-mentioned general result to show that Chowla's conjecture on correlations of the Moebius function holds on average and we strengthened previous results on patterns in the Liouville function (a close cousin of the Moebius function).

I will explain in detail the meaning, motivation and origins of the above problems and conjectures.

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