# BI-CO MATHEMATICS COLLOQUIUM


## David Shea Vela-Vick
University of Pennsylvania


## *"Triple linking numbers, ambiguous Hopf invariants and integral formulas for three-component links"*

## Monday, March 30, 2009

Talk at 4:15 p.m. – KINSC H109
Tea at 4:00 p.m. – KINSC H208, Math Lounge


**Abstract**: Three-component links in the 3-dimensional sphere were classified up to link homotopy by John Milnor in his senior thesis. A complete set of invariants is given by the pairwise linking numbers p, q and r of the components, and by the residue class of one further integer mu, which is well-defined modulo the greatest common divisor of p, q and r.

To each such link L we associate a geometrically natural characteristic map g_L from the 3-torus to the 2-sphere in such a way that link homotopies of L become homotopies of g_L. Maps of the 3- torus to the 2-sphere were classified up to homotopy by Pontryagin in 1941. A complete set of invariants is given by the degrees p, q and r of their restrictions to the 2-dimensional coordinate subtori, and by the residue class of one further integer nu, an "ambiguous Hopf invariant" which is well-defined modulo twice the greatest common divisor of p, q and r.

We show that the pairwise linking numbers p, q and r of the components of L are equal to the degrees of its characteristic map g_L restricted to the 2-dimensional subtori, and that twice Milnor's mu-invariant for L is equal to Pontryagin's nu-invariant for g_L.

When p, q and r are all zero, the mu- and nu-invariants are ordinary integers. In this case we use J. H. C. Whitehead's integral formula for the Hopf invariant, adapted to maps of the 3-torus to the 2- sphere, to provide an explicit integral formula for nu, and hence for mu.


**HAVERFORD COLLEGE**