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“Triple linking numbers, ambiguous Hopf invariants and integral formulas for three–component links”

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Talk at 4:15 p.m. – KINSC H109
Tea at 4:00 p.m. – KINSC H208, Math Lounge

Abstract: Three–component links in the 3–dimensional sphere were classified up to link homotopy by John Milnor in his senior thesis. A complete set of invariants is given by the pairwise linking numbers p, q and r of the components, and by the residue class of one further integer mu, which is well–defined modulo the greatest common divisor of p, q and r.

To each such link L we associate a geometrically natural characteristic map g_L from the 3–torus to the 2–sphere in such a way that link homotopies of L become homotopies of g_L. Maps of the 3–torus to the 2–sphere were classified up to homotopy by Pontryagin in 1941. A complete set of invariants is given by the degrees p, q and r of their restrictions to the 2–dimensional coordinate subtori, and by the residue class of one further integer nu, an "ambiguous Hopf invariant" which is well–defined modulo twice the greatest common divisor of p, q and r.

We show that the pairwise linking numbers p, q and r of the components of L are equal to the degrees of its characteristic map g_L restricted to the 2–dimensional subtori, and that twice Milnor's mu–invariant for L is equal to Pontryagin's nu–invariant for g_L.

When p, q and r are all zero, the mu– and nu–invariants are ordinary integers. In this case we use J. H. C. Whitehead's integral formula for the Hopf invariant, adapted to maps of the 3–torus to the 2–sphere, to provide an explicit integral formula for nu, and hence for mu.

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