BI-CO MATHEMATICS COLLOQUIUM

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"Lengths and Crossing Numbers of Knotted Ropes and Strips"

Monday, April 28, 2008
Tea at 4:00 p.m. – Math Lounge, KINSC H208
Talk at 4:15 p.m. – KINSC Hilles 109

Abstract: Abstract: About a decade ago, biophysicists observed an approximately linear relationship between the combinatorial complexity of knotted DNA and the distance traveled in gel electrophoresis experiments [1]. Modeling the DNA as tightly knotted rope of uniform thickness, it was suggested that lengths of such tight knots (rescaled to have unit thickness) would grow linearly with crossing numbers, a simple measure of knot complexity. It turned out that this relationship is more subtle: some families of knots have lengths growing as the ¾ power of crossing numbers, others grow linearly, all powers between 3/4 and 1 can be realized as growth rates, and it could be proven that that the power cannot exceed 2 [2-5]. It is still unknown whether there are families of tight knots whose lengths grow faster than linearly with crossing numbers, but the largest power has been reduced to 3/2 [6]. We will survey these and more recent developments in the geometry of tightly knotted ropes, as well as some other physical models of knots as flattened ropes or strips which exhibit similar length versus complexity power laws [7].

References:
[1] Stasiak A, Katritch V, Bednar J, Michoud D, Dubochet J
"Electrophoretic mobility of DNA knots" Nature 384 (1996) 122
[2] Cantarella J, Kusner R, Sullivan J "Tight knot values deviate from linear relation" Nature 392 (1998) 237
[3] Buck G "Four-thirds power law for knots and links" Nature 392 (1998) 238
[4] Buck G, Jon Simon "Thickness and crossing number of knots" Topol. Appl. 91 (1999) 245
[5] Cantarella, J, Kusner R, Sullivan J "On the minimum rope length of knots and links" Invent. Math. 150 (2002) 257
[6] Diao Y, Ernst C, Yu X "Hamiltonian knot projections and lengths of thick knots" Topol. Appl. 136 (2004) 7
[7] Diao Y, Innis E, Kusner R [work in progress]

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