BI-CO MATHEMATICS COLLOQUIUM

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"Counting Lattice Paths that Avoid a Boundary"

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Talk at 4:15 p.m. – KINSC H109
Tea at 4:00 p.m. – KINSC H208, Math Lounge

Abstract: How many ways are there to get from (0,0) to (a,b) taking only unit steps east or north and never crossing the line $x = y$? The simple formula for this number can be proved by a beautifully simple combinatorial argument called the "reflection principle," introduced by Andre in 1887. If the line $x = y$ is replaced by the line $x = ky$ for some integer k, then there is still a simple formula, also known since 1887, but apparently a proof in the spirit of the reflection principle (using a "rotation principle") was not published until the 21st century, by Goulden and Serrano.

What if $k = s/t$ is a non-integer rational? Then nothing simpler than the determinant of a matrix of binomial coefficients is known. However, we show that if we replace $x = ky$ by a staircase boundary with steps that are s wide and t high, then a simple formula exists, at least if the endpoints of the path are carefully chosen. We give two combinatorial proofs, one in the spirit of the classic "Raney lemma" and one using an "interchange principle." It remains an open question whether our results can be further generalized, and time permitting we will mention a tantalizing result that suggests that more theorems lie waiting to be discovered.

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