BI-CO MATHEMATICS COLLOQUIUM

David Cervone
Union College

"The Hypercube and Hypersphere: Breaking them Down and Building them Up"

Monday, October 8, 2007

Talk at 4:15 p.m.– KINSC H109
Tea at 4:00 p.m. – Math Lounge, KINSC H208

Abstract The familiar decomposition of a sphere in 3-space into two hemispheres can also be applied to the hypersphere in 4-space: while in 3-space we use two disks joined along their boundaries, the analog in 4-space is to join two solid spheres by gluing them along their bounding spheres. There is, however, another interesting decomposition of the hypersphere into two congruent parts. It turns out that the hypersphere can be broken down into two linked solid tori glued together along their surfaces. In this talk, we analyze this decomposition by first looking at the hypercube as a polyhedral model of the hypersphere, and develop the corresponding breakdown within it. To return to the hypersphere, we "smooth out" the polyhedral version to obtain the desired decomposition of the 3-sphere in 4-space. This leads to both beautiful mathematics and beautiful images, with connections to a variety of mathematical fields, from dynamical systems to complex analysis.

HAVERFORD COLLEGE