

# B I - C O M A T H E M A T I C S C O L L O Q U I U M

Juliana Belding (BMC '01)  
University of Maryland

*“Curves, Cryptography and Calculus: A  
Weil pairing on Elliptic Curves over the  
Dual Numbers”*

**Monday, December 3, 2007**

Talk at 4:15 – Park 338  
Tea at 3:45 – Park 355, Math Lounge

**Abstract:**

First proposed in 1985, elliptic curve cryptosystems are now widely used by government and industry and represent an active area of research for mathematicians. I will introduce elliptic curves and the basic idea of elliptic curve cryptography, and show how the Weil pairing on the points of order  $n$  of an elliptic curve over a field  $K$  with characteristic prime to  $n$  is used to attack the security of certain cryptosystems.

I will then discuss my work with elliptic curves over the dual numbers of  $K$ , the set of elements of the form  $a + be$  where  $a, b$  are in  $K$  and  $e^2 = 0$ . These numbers behave like numbers with extra “derivative” information—their arithmetic mimics the basic rules of calculus. I will show how in working over the dual numbers, we can extend the Weil pairing to points of order  $p$  where  $p$  is relatively prime to the characteristic of  $K$ . In doing so, we recover an attack on trace one elliptic curves, which was first discovered in 1999.

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