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Manifolds of Positive Sectional Curvature

Sectional curvature is perhaps the most fundamental geometric invariant in differential geometry. It is an attempt by mathematicians to characterize our intuitive notion of the difference between something which is "flat" and something which "bends" in space. Under this scheme geometers tend to study either Riemannian manifolds of nonpositive sectional curvature or Riemannian manifolds of nonnegative sectional curvature.

The study of manifolds of nonpositive sectional curvature has enjoyed a good degree of success over the years. In particular, the theory of manifolds of negative sectional curvature is quite rich and examples are plentiful. However, the same cannot be said of manifolds of nonnegative or positive sectional curvature. Specifically, there are very few examples of manifolds of positive sectional curvature in the literature and there are few known topological obstructions to having positive sectional curvature. In short, the subject is fairly wide open.

In this talk we will introduce curvature and discuss some theorems which demonstrate the interplay between the curvature of a manifold and its topology. We also aim to familiarize the audience with some of the basic examples of manifolds of positive sectional curvature (e.g., the CROSSes) and discuss the role that Riemannian submersions play in constructing such spaces.