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How Many Ways Can You Parallel-Park a Unicycle?

You can describe the configuration of a unicycle on a sidewalk by three coordinates: two position coordinates x and y for where the wheel comes into contact with the ground and one angle coordinate t that describes the angle that the direction the wheel makes with the x axis. At a given point (x,y,t) , the instantaneous motions of the unicycle (if we do not want to scrape the tire by trying to move sideways) are constrained to moving in the direction the wheel is pointing, turning the wheel without moving forward, or some combination of the two. As you pedal around, you trace out a path in (x,y,t) -space that obeys the constraints at every point.

The system of constraints at every point in (x,y,t) -space is an example of a "contact structure," and a path that obeys the constraints is a "Legendrian curve." If the curve returns to its starting point, then it is called a "Legendrian knot." A central question in the theory of Legendrian knots is: how can you tell two Legendrian knots apart? How many are there? In other words, how many ways are there to parallel park your unicycle?

There will NOT be a practical demonstration.