

# Aging, Secular Stagnation and the Business Cycle

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## Abstract

This paper studies the macroeconomic consequences of the aging US population in a lifecycle model with endogenous labor supply. I input to the model the trends in mortality and fertility observed between 1940 and 2015, and projected to 2070. The model accounts well for the decline in the real interest rate from 1980, slower output and productivity growth after 2009, and the fall in the employment-population ratio from 2000. I show that the aggregate consequences of demographic changes in the lifecycle model are well approximated by a representative agent RBC model with trends in productivity, the marginal utility of consumption, and the labor wedge. I incorporate pricing frictions and the zero lower bound on nominal interest rates into this framework and study the model's business cycle implications. I find that when demographic mechanisms are taken into account, the zero lower bound is significantly more likely to bind between 2010 and 2025.

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# 1 Introduction

A number of weak macroeconomic trends have emerged since the Great Recession. Average output and productivity growth has fallen, the labor force participation rate has remained below pre-recession levels, while the Federal Funds rate has been held at its zero lower bound by the Federal Reserve since 2009—an unprecedented length of time. The persistence of these declines following the financial crisis has generated a debate about whether the US is in a ‘secular stagnation’ characterized by *sustained* low growth and negative real interest rates, as suggested by [Summers \(2014\)](#), or whether weak macroeconomic performance is driven by *temporary* ‘headwinds’, such as a long process of deleveraging following the financial crisis ([Rogoff, 2015](#)).

The average age of the US population has been steadily increasing over the past half-century. Changes in longevity and to fertility are driving this aging process. Health improvements have decreased mortality rates and increased life expectancy for all generations. A transitory but large increase in fertility following the second world war gave rise to the ‘baby boomer’ cohort that entered the labor market between the 1960s and 1980s. This change in fertility has caused substantial changes in the composition of the population by age groups over time. Taken together, the aging of the population and large variation in the age-composition of the population is expected to have a significant effect on the US economy ([Gagnon et al., 2016](#); [Maestas et al., 2016](#); [Wong, 2015](#); [Bloom et al., 2001](#); [Auerbach and Kotlikoff, 1987](#)).

In this paper, I argue that a full consideration of the aging population can largely explain the medium-term movements in real interest rates, output and productivity growth, and in employment-population ratios. These trends, in turn, have an important effect on business cycle fluctuations, particularly in light of the zero lower bound on nominal interest rates. I first show that the observed medium-term macroeconomic trends can be largely replicated in a dynamic lifecycle model parameterized to observed demographic changes. I then embed demographic trends into a novel business cycle framework incorporating the zero lower bound on nominal interest rates. This framework allows me to study key questions in the secular stagnation debate by separating the trend component from transitory fluctuations.

The paper explores two broad questions. First, to what extent can demographics explain the *trends* in key macroeconomic variables that have emerged in the US over the medium term? In particular, to what extent can the aging of the population explain declining real interest rates, output and productivity growth, and employment-to-population ratios? How long should we expect demographic trends to affect the economy? Do demographic trends predict an extended period of economic weakness, in line with the secular stagnation hypothesis? The answers to these questions are not immediately clear as demographic trends may generate general equilibrium effects that work against the secular stagnation hypothesis. For example, as the pool of workers contracts as the baby boomers retire, we might expect higher real wages and spending, which in equilibrium would push up the real interest rate and investment.<sup>1</sup> To account for and disentangle these effects to answer the questions just posed, I use a quantitative lifecycle model with endogenous labor supply. The model is calibrated to lifecycle data and to observed demographic changes.

The second issue I explore is: how important, quantitatively, are these demographic trends for the transmission of business cycle shocks to the rest of the economy? In particular, how does the likelihood that the nominal interest rate hits the zero lower bound change with the demographic state? This question is motivated by the results from the first part of the paper, which ascribe a significant role for demographics in explaining the decline in the real interest rate. I answer this in two steps, by first connecting the lifecycle model I use to the real business cycle (RBC) model of [Kydland and Prescott \(1982\)](#) using anticipated shocks to preferences and technology (following [Maliar and Maliar, 2003](#)). In the second step, I add pricing frictions and monetary policy to the aggregated model, and use a methodology that accommodates the anticipated demographic shocks in the solution to estimate the business cycle model. The efficient setup of the methodology provides a new way to link long-run trends with the business cycle in the spirit of [Comin and Gertler \(2006\)](#), to study and quantify the implications of demographics for monetary policy in the presence of the zero lower bound.<sup>2</sup>

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<sup>1</sup>As suggested by Charles Goodhart, Pratyancha Pardeshi and Manoj Pradhan, in ‘Workers vs Pensioners: The Battle of Our Time’, *Prospect Magazine*, November 12, 2015.

<sup>2</sup>[Wong \(2015\)](#) showed that the transmission of monetary policy shocks depends on the demographic state, through household mortgage refinancing.

Turning to the structure of the paper, I first study in [Section 2](#) the effects of demographic changes with an empirical analysis of the extent to which changes in the age-composition of the population can explain declines in aggregate labor force participation rates and real gross domestic product growth rates. Holding constant labor force participation rates by age and varying age-population shares suggests that simply changing the age-composition of the population can explain about one-third of the observed decline in the labor force participation rate from 2008 to 2015, consistent with empirical studies ([Aaronson et al., 2014](#)). However, I also find that the proportion explained depends on the year that the age-profile for employment is held constant, suggesting that labor supply incentives are changing over time. I also show in [Section 2](#) that there are strong correlations between measures of growth and the population age-composition using a cross-country panel dataset. I find that countries with older populations on average have lower rates of growth in real output per capita. These results are consistent with the findings of [Maestas et al. \(2016\)](#) and [Feyrer \(2007\)](#). Taken together, the empirical evidence motivates the use of the structural model to capture changes in labor supply incentives during the demographic transition and to study why output and productivity growth varies with demographic trends.

In [Section 3](#), I describe the lifecycle model that I use to study the role of demographic changes. The model features 80 overlapping generations, endogenous labor supply and age-specific human capital. There are two demographic factors that are the only exogenous sources of variation in the model. First, workers have age-specific and time-varying mortality rates, which are matched to observed and projected mortality profiles. Second, the size of the initial cohort each period is exogenous and chosen to match observed changes in the relative size of the the 16-year-old cohort. Fully anticipated paths for these two demographic trends enter the model, generating predictions for the endogenous variables of the model.

In [Section 4](#), I calibrate the model and discuss the three main results on the trend paths of key macroeconomic variables that the lifecycle model speaks to. First, I quantify how the real interest rate—the marginal product of capital—changes over the demographic transition. I find that demographic trends imply that the real interest rate has declined by 1.5 percentage

points from 1980 to 2015. This compares to a 1.9 percentage point decline in the real interest rate from 1980 to 2015, where the observed decline is computed, in contrast with much of the literature on secular stagnation, using observed capital-output ratios. The two exogenous demographic changes have very different effects on the real interest rate. The gradual decline in mortality rates causes a gradual compositional shift towards an older population: longer expected lifetimes generate an increase in the relative demand for savings and capital for retirement consumption. This component contributes about a 1 percentage point decline in the long-run value of the real interest rate. The temporary fertility shocks, however, cause a large oscillation around the path implied by the decline in mortality rates. The oscillation is primarily caused by changes in the relative size of the workforce. When the workforce is relatively young, aggregate hours supplied is high relative to aggregate savings and capital, pushing up the marginal return to capital. As the cohort ages, they accumulate savings for retirement, pushing down the marginal return to capital. These workers then begin to withdraw from the labor market, rapidly pushing down the marginal return to capital. I project the real interest rate to bottom out around 2023 and remain low beyond 2030.

Second, I find that my model predicts that the *growth rate* of labor productivity—output per hour worked—has declined by 0.5 percentage points from 1990 to 2015. This compares to the observed decline of 1 percentage point in average productivity growth over the same period. To shed light on why productivity growth declines, I decompose the model path into three components: hours, capital, and labor quality. Accumulated hours supplied is high when the baby boomer generations enter the workforce, pushing up total output growth. Because capital is slower to increase as compared to the increase in labor supplied, labor productivity growth declines. Over time, the second dimension—the change in aggregate savings and capital—increases output and productivity growth over the demographic transition, mainly due to the strong savings demand from the baby boomer generations. The third component is that the *quality* of labor changes. Workers undergo substantial changes in their marginal productivity as they age, as reflected in the hump-shaped age-earnings profile. Demographic trends change the mix of workers over the age-productivity curve. As the baby boomer

generation enters the model, there is an increase, and subsequent decrease, in the accumulation of human capital and therefore in output and productivity growth.

Third, I find that the employment-population ratio has declined by 1.75 percentage points in the model from 1990 to 2015, and can be expected to decline by a further six percentage points over the next 20 years, to 2035. This compares to the observed decline of 3 percentage points from 1990 to 2015. Existing empirical studies also predict declines in the labor force participation rate as a consequence of demographic changes (see, for example, [Aaronson et al., 2014](#)). I add to these studies and show that the lifecycle model I use makes clear the relative importance of general equilibrium effects in explaining changes in the employment-population ratio since 2000. The demographic trends imply a shrinking labor force relative to the capital stock accumulated for retirement consumption. The equilibrium effect on wages incentivizes workers to stay in the workforce, but is not enough to offset the compositional effect. In discussing my results, I show how the response of labor force participation rates vary with equilibrium changes in wages, showing that they do a good job in explaining the data, particularly for older workers.

The striking results on the role of demographic trends imply that at a point in time, the underlying structure of the economy changes significantly over the demographic transition. This motivates an analysis of how demographic trends interact with shocks that drive the business cycle. To this end, in [Section 5](#), I discuss how heterogeneity in the lifecycle model can be summarized in the real business cycle (RBC) framework with slow-moving, anticipated trends to the level of technology, the marginal utility of aggregate consumption, and the labor wedge. These trends vary over time with demographic changes, and can be easily constructed from exogenous demographic information, and assumptions about the age-productivity profile and the disutility of providing labor over the lifecycle.<sup>3</sup>

In [Section 6](#), I then describe and use a flexible methodology that incorporates these demographic trends as anticipated changes in the parameters of the real business cycle model.<sup>4</sup> I add pricing frictions and the zero lower bound to the business cycle model, and set

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<sup>3</sup>An insight from this procedure is that exogenous demographic trends simultaneously affect aggregate shocks in the standard RBC framework, which many studies estimate as orthogonal processes.

<sup>4</sup>More generally, my paper provides a methodology for incorporating underlying long-run demographic

it to a quarterly frequency.<sup>5</sup> I estimate the parameters of the monetary policy rule and the parameters of four exogenous shock processes that drive business cycle fluctuations, when demographics are taken into account and when the zero lower bound binds using, as an observable, the length of time that the zero lower bound is expected to bind each period. I repeatedly simulate the estimated quarterly model and study the distribution of the simulated path for the nominal interest rate over time. I find that the zero lower bound is significantly more likely to bind from 2010 to 2025 when demographic trends are accounted for in the solution to the model, finding that for those time periods the nominal interest is negative for between 30% and 40% of the density of the simulated paths. Imposing the zero lower bound in the simulations of the model increases this proportion.

## Related literature

Here, I discuss three strands of the literature that this paper relates to.

The first literature covers the ‘secular stagnation’ debate proposing that the United States, and much of the developed world, is facing a permanently depressed real interest rate (Hamilton et al., 2015; Rogoff, 2015; Rachel and Smith, 2015; Bernanke, 2015; Summers, 2014; Eggertsson and Mehrotra, 2014). Key papers in this literature suggest that these trends have been partly caused by an aging population. In this paper, I study the extent to which demographics can explain a decline in the real interest rate and compare it to a measure in the data that is closely related to the model, namely the marginal product of capital constructed from capital-output ratios as reported by the Bureau of Labor Studies.<sup>6</sup>

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trends in the estimation of business cycle models at a quarterly frequency. The procedure makes progress towards addressing criticisms of DSGE models by Blanchard (2016). In particular, I bring external information (in this case, about demographic changes) to an otherwise standard DSGE model suitable for estimation.

<sup>5</sup>In specifying and solving the model with the zero lower bound, the methodology can accommodate calendar-based forward guidance as a tool of monetary policy following my earlier work in Jones (2015a).

<sup>6</sup>Different studies use different measures of the real interest rate. Eggertsson et al. (2016) measure the real interest rate using Treasury yields and inflation rates, finding that they trend down, while Gomme et al. (2011) report that the return to capital as measured by the income generated from capital relative to the size of the capital stock has not declined as much as real returns computed off Treasury yields, and has instead recovered to its pre-crisis levels. I use as my measure of the real interest rate the return to capital computed from the capital-output ratio reported by the Bureau of Labor Studies Multifactor Productivity program and used in Fernald (2015). This series is most closely related to the model quantity that I compare the data to. That series shows a clear declining trend. It would be an interesting area of future work to study why these different estimates of the return to capital have diverged.

A number of other papers tie to the secular stagnation literature by studying the decline in long-run growth and its consequences (Gordon, 2016; Antolin-Diaz et al., 2014; Fernald, 2015; Anzoategui et al., 2015; Christiano et al., 2015). In this paper, I consider how demographics relate to measures of output and productivity growth, through changes in aggregate hours and capital and a distributional channel with a changing age-composition of the workforce over the age-productivity curve when labor supply is endogenous.<sup>7</sup>

The second literature this paper relates to is a substantial body of work that studies the macroeconomic implications of demographic changes in structural overlapping generations models (an incomplete list includes the work of Aksoy et al., 2015; Carvalho et al., 2015; Fujita and Fujiwara, 2014; Backus et al., 2014; Rios-Rull, 2007; Krueger and Ludwig, 2007; Kulish et al., 2006; Abel, 2003; Bloom et al., 2001; De Nardi et al., 1999; Gertler, 1999; Auerbach and Kotlikoff, 1987). These papers focus on a range of long-term consequences of demography, including on the solvency of social security systems, and for studying international capital flows. There are also a number of empirical studies of the macroeconomic implications of demographic changes (see, for example, Bloom et al., 2011). My paper builds on the insights of the recent empirical work connecting demographic trends to output growth (Maestas et al., 2016), productivity growth (Feyrer, 2007), and the labor force participation rate (Aaronson et al., 2014), and focuses on jointly explaining these post-crisis trends in a lifecycle model.

The third strand of literature the paper relates to are studies of how the propagation of transitory shocks can change as an economy undergoes structural changes, either anticipated or unanticipated (Canova et al., 2015; Wong, 2015; Kulish and Pagan, 2012; Jaimovich and Siu, 2012; Fernández-Villaverde et al., 2007). This paper builds on this work by providing a methodology for explicitly accounting for demographics in the estimation of a business cycle model. By incorporating these demographic trends as anticipated shocks in a business cycle model, the framework in this paper can be used to study a number of interesting questions; I

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<sup>7</sup>Understanding the extent to which this component is driving a decline in labor productivity is important because a number of papers take the decline in total factor productivity (TFP) growth as permanent and exogenous. For example, Eggertsson et al. (2016) calibrate a permanent decline in this component of 1.4 percentage points, based on observed TFP changes from 1990 to 2010, and on top of that decline, include age-specific human capital and changing demographics. Antolin-Diaz et al. (2014) find that TFP growth has declined since 2000 by about the same magnitude. Christiano et al. (2015) estimate a permanent decline in TFP as part of their model of the Great Recession.

focus on two: how demographic trends affect the transmission of transitory shocks, and how demographics affect the likelihood that the zero lower bound will bind. To my knowledge, my paper is the first to quantify how much *more* the zero lower bound will bind in the presence of a demographic-induced decline in the real interest rate.

I close the literature review with a discussion of three recent and closely related papers. [Eggertsson et al. \(2016\)](#) study a number of possible explanations for a decline in the real interest rate, through the lens of a quantitative lifecycle model. One of those factors is demographic changes to longevity and fertility, as I study. I relate to their work by studying a larger set of trends in important variables—output and productivity growth, and employment-population ratios—in understanding more broadly the importance of demographic trends. My model additionally incorporates labor supply decisions, which I find vary significantly by age and over time. I find that the endogenous response of labor supply in the lifecycle framework can temper some of the fluctuations in the marginal return to capital, and therefore the real interest rate and aggregate total factor productivity growth. Finally, in my framework, I show that demographics can account for almost all of the magnitude of the decline in the real interest rate from 1990 to 2015, as measured by the marginal product of capital computed using observed capital-output ratios.

[Carvalho et al. \(2015\)](#) consider the relationship between real interest rates and demographics in a model where individuals face, each period, an age-independent probability of death, and an age-independent probability of exiting the labor force. The paper identifies shocks to these two probabilities to match life expectancy and age-dependency ratios, over time. My paper differs in two important ways: first, I model the full lifecycle dimension where mortality rates are age-specific, and second, I model labor supply, so that the dependency ratio emerges as an endogenous object. I find a similar permanent decline in the real interest rate of about 1.5% over the 1990 to 2015 period, but for different reasons. Rather than the permanent, and steady, increase in life-expectancy that is important in their framework, I find that changes in the age-distribution of the population is key: in particular, I find a significant role for the aging of the baby boomer generation. In addition, I find that demographic forces do a good

job at explaining the increase over time in labor supply by those on the edge of retirement and, simultaneously, the slight decline in hours supplied by those entering the labor market.

The final paper I discuss relates to the second part of this paper, on how the transmission of business cycle shocks varies with the prevailing demographic profile. [Wong \(2015\)](#) looks at how the impact of monetary policy shocks differs with the age-profile of the economy, finding, empirically, that older people have lower marginal propensities to consume, with the key channel being the lower sensitivity of older people to mortgage refinancing. [Wong \(2015\)](#) estimates that this channel alone implies a substantially smaller aggregate response of consumption to a monetary policy shock. My paper builds on this work by providing a tractable way to incorporate demographic trends into a business cycle model to study the interaction between demographics and a suite of business cycle shocks. This new approach is able to account for general equilibrium effects in the transmission of transitory shocks.

## 2 Empirical overview of consequences of demographic changes

I start with a simple empirical overview of the aggregate effects of demographics to motivate the use of the lifecycle model for understanding the relationship between demographics and macroeconomic trends. The first analysis is a shift-share exercise on age-specific labor force participation rates and a shift-share exercise of the change in age-specific labor quality to study how the age-composition of the population drives aggregate participation rates and labor productivity. The second analysis focuses on the relationship between real gross domestic product growth and population shares by age.

### 2.1 Changes in the age-composition of the population

In the first exercise, I fix the age-specific labor force participation rates at their values observed in a time period  $\tau$  and then vary the population-shares of each cohort as observed to compute a counterfactual series for the aggregate participation rate:

$$\frac{\ell_t}{n_t} = \sum_{s=16}^{95} \ell_\tau^s \frac{n_t^s}{n_t}, \quad (1)$$

**Table 1:** Shift-share analysis of population changes

| $l_\tau^s$ profile                                | Change, percentage points |              |              |              | Level, % |       |       |
|---------------------------------------------------|---------------------------|--------------|--------------|--------------|----------|-------|-------|
|                                                   | 1948 to 1969              | 1970 to 1995 | 1996 to 2007 | 2008 to 2015 | 1970     | 1990  | 2015  |
| <i>A. Participation rate, Equation (1)</i>        |                           |              |              |              |          |       |       |
| $\tau = 1980$                                     | -3.05                     | -0.34        | -0.28        | -1.63        | 62.09    | 61.96 | 59.60 |
| $\tau = 1990$                                     | -3.34                     | -0.16        | -0.35        | -1.83        | 65.14    | 65.09 | 62.56 |
| $\tau = 2000$                                     | -2.99                     | -0.37        | -0.15        | -1.67        | 63.80    | 63.53 | 61.41 |
| $\tau = 2010$                                     | -3.47                     | 0.08         | -0.03        | -1.38        | 65.11    | 65.31 | 63.59 |
| Data                                              | 0.90                      | 6.20         | -0.80        | -3.40        | 59.60    | 65.90 | 63.30 |
| <i>B. Labor quality growth rate, Equation (2)</i> |                           |              |              |              |          |       |       |
| $\tau = 1980$                                     | -0.36                     | 0.32         | -0.22        | 0.07         | -0.20    | 0.32  | -0.09 |
| $\tau = 1990$                                     | -0.36                     | 0.32         | -0.22        | 0.07         | -0.20    | 0.33  | -0.09 |
| $\tau = 2000$                                     | -0.35                     | 0.32         | -0.22        | 0.06         | -0.19    | 0.32  | -0.10 |
| $\tau = 2010$                                     | -0.32                     | 0.32         | -0.25        | 0.05         | -0.18    | 0.29  | -0.12 |
| Data                                              | 0.01                      | 0.60         | -0.05        | -0.26        | 0.08     | 0.70  | 0.39  |

Notes: The labor quality data series is constructed by [Aaronson and Sullivan \(2001\)](#), and used in [Fernald \(2015\)](#). It is HP-filtered.

where  $\ell_t/n_t$  is the total supply of labor hours over the total available hours, and  $\ell_\tau^s$  is the age  $s$  participation rate measured in period  $\tau$ , and obtained from the Current Population Survey. The decomposition is repeated for  $\ell_\tau^s/n_\tau^s$  profiles observed each decade between 1980 and 2010.

Panel A of [Table 1](#) presents the counterfactual total participation rate. It shows that demographics can replicate roughly half of the observed changes in the participation rate since 1996. In particular, changes in the size of the population appear to be able to capture some of the increase in the participation rate between 1970 and 1995, with the unexplained part driven by the increase in female labor force participation not captured when age-specific participation rates are held constant. Compositional changes then appear to be able to explain about one-third of the decrease in the overall labor force participation rate between 2008 to 2015, which is the period when the first cohorts of the baby boomer generation started to withdraw from the labor market.

This compositional analysis suggests an important role for demographics in explaining actual changes in the participation rate, particularly for the post-1990 period. However, the

degree to which it explains the decline varies with the profile for the age-specific participation rates. Using the age-specific participation rates observed in 1990 assigns about a 30% larger explanation for the decline in the total participation rate to changes in the age-composition of the workforce, as compared to the decline computed off the 2010 age-specific participation rate. At first glance, this suggests that workers' labor supply decisions are offsetting some of the purely compositional effect of population changes on the aggregate labor force participation rate. The model I use in the next section endogenizes the labor supply decision at each age, accounting for how labor supply reacts to equilibrium changes in wages and asset holdings.

Panel B of [Table 1](#) presents a shift-share analysis on changes in labor quality and compares them to a measure constructed by [Fernald \(2015\)](#). Labor quality is measured by the difference between aggregate effective labor supplied and raw hours. It is a measure of the average productivity of the workforce and can change with the composition of the workforce by age. I construct labor quality with the following ratio:

$$LQ_t = \frac{\sum_{s=16}^{95} z^s \ell_{\tau}^s \frac{n_t^s}{n_t}}{\sum_{s=16}^{95} \ell_{\tau}^s \frac{n_t^s}{n_t}}, \quad (2)$$

where  $z^s$  is the relative productivity of a worker of age  $s$ , calibrated to Census and American Community Survey extracts.<sup>8</sup> This ratio is constructed for different profiles for the age-specific participation rate  $\ell_{\tau}^s$ . I compare this ratio to a measure of labor quality at each point in time is constructed by [Aaronson and Sullivan \(2001\)](#) and [Fernald \(2015\)](#), who use Mincerian wage regressions with Census data and individual characteristics, including education and tenure.

The measure of labor quality reported by [Fernald \(2015\)](#) shows that there is a humped-shaped profile for the growth rate of labor quality over 1950 to 2015, with labor quality initially growing slowly, before accelerating between 1970 and 1995, after which it becomes negative. With labor quality being a fraction of labor productivity, this suggests that changes to the composition of workers became a drag on labor productivity in the 2000s. Holding constant labor supply participation rates  $l_t^s$  shows that changes in the size of relative cohorts alone can account for a large part of the decline in labor quality in the data, and therefore,

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<sup>8</sup>This calibration is discussed in more detail when the lifecycle model is calibrated.

labor productivity.

The results in [Table 1](#) therefore ascribe an important role to demographic trends in explaining changes in labor quality through compositional variation in the age-structure of the workforce. Empirically, changes in labor quality are an important part of changes in labor productivity ([Fernald, 2015](#)). However, labor quality is only one component potentially driving labor productivity. Changes in total labor supply and aggregate capital will affect output with varying intensity over the demographic transition. The increase in aggregate hours will be particularly strong following a sustained increase in fertility, as many young will enter the labor force at the same time. For investment, the consumption smoothing motive can be expected to dampen the growth rate of capital when the mass of baby boomers are young and are borrowing for consumption, but see it accelerate as they build up savings for retirement. The lifecycle model will be used to decompose productivity growth into its components over the demographic transition.

## 2.2 Output growth

How is output growth correlated with a country's age structure? To answer this, [Figure 1](#) plots, for eight developed countries over the years 1950 to 2015, real GDP growth against a country's population share for different population groups<sup>9</sup>. The relationship between the population share for different age groups and real GDP growth shows that a country's real GDP growth rate tends to decline on average as a country ages. Furthermore, the slope of the regression lines becomes increasingly negative in the age of the population share, suggesting that the decline in growth accelerates as the population ages.

[Table 2](#) presents the results of a panel regression of real GDP growth per capita on 10-year population shares, using the same set of countries as was used to construct [Figure 1](#). These results show the correlation observed in the figure: that countries with young populations tend to grow faster, and that the decline in growth is higher for older societies. The pattern can be driven by how demographics changes the supply of labor and the supply of capital

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<sup>9</sup>The population data is extracted from the United Nations Population Division. The output data is extracted from the Federal Reserve Economic Database.

**Table 2:** Dependent variable is real GDP growth per capita

| Variable          | (1)                | (2)                 | (3)               | (4)                 | (5)                 | (6)                 |
|-------------------|--------------------|---------------------|-------------------|---------------------|---------------------|---------------------|
| Age 20-29 % share | 0.246**<br>(0.039) |                     |                   |                     |                     |                     |
| Age 30-39 % share |                    | 0.369**<br>(0.063)  |                   |                     |                     |                     |
| Age 40-49 % share |                    |                     | 0.018<br>(0.069)  |                     |                     |                     |
| Age 50-59 % share |                    |                     |                   | -0.299**<br>(0.072) |                     |                     |
| Age 60-69 % share |                    |                     |                   |                     | -0.714**<br>(0.085) |                     |
| Age 70-79 % share |                    |                     |                   |                     |                     | -0.880**<br>(0.084) |
| Intercept         | -0.024*<br>(0.009) | -0.048**<br>(0.013) | 0.027*<br>(0.013) | 0.079**<br>(0.012)  | 0.122**<br>(0.011)  | 0.104**<br>(0.007)  |
| Country FE        | ✓                  | ✓                   | ✓                 | ✓                   | ✓                   | ✓                   |
| N                 | 408                | 408                 | 408               | 408                 | 408                 | 408                 |
| R <sup>2</sup>    | 0.090              | 0.079               | 0.001             | 0.041               | 0.149               | 0.214               |

Notes: The countries included in the regressions are: Australia, Canada, France, Italy, Japan, New Zealand, the United Kingdom, and the United States, over the years 1950 to 2015. Standard errors are in parentheses. The data are from the United Nations Population Division and the Federal Reserve Economic Database.

(Maestas et al., 2016), and by productivity (Feyrer, 2007). I use the structural model in the next section to decompose exactly how output growth changes over the demographic transition that is observed in the US. The structural model can generate the correlations and elasticities of output growth to the age-composition observed in [Table 2](#).

### 3 Lifecycle model of the demographic transition

In this section, I develop an annual overlapping generations model with endogenous labor supply. The two exogenous variables in the model are the two demographic trends of increasing longevity and declining fertility.<sup>10</sup>

<sup>10</sup>To keep the focus on these two demographic trends, the model does not feature exogenous technology growth or a constant growth rate of the population.

### 3.1 Households

The economy is populated by  $T$  overlapping generations of mass  $n_t^s$ , comprised of identical members of age  $s$ . The size  $n_t^s$  is measured at the start of period  $t$ . I abstract from trend population growth, so that in the absence of exogenous increases in the size of the incoming population, the initial population size is normalized to  $n_t^0 = 1$ . Each generation lives for a maximum of  $T$  periods, so that the age range of an individual is 0 to  $T - 1$ . The total size of the population at time  $t$  is therefore:

$$n_t = \sum_{s=0}^{T-1} n_t^s.$$

Each period, a fraction of age  $s$  individuals die with an exogenous age-specific mortality probability  $\gamma_t^s$ :

$$n_{t+1}^{s+1} = (1 - \gamma_t^s)n_t^s.$$

These mortality rates vary exogenously over time. In particular, permanent decreases in mortality rates reflect increases in longevity due to, for example, health improvements.

An individual of age  $s$  has the period utility function  $u(c_t^s, \ell_t^s)$  and chooses consumption  $c_t^s$ , the intensive margin of labor  $\ell_t^s$  and savings  $a_t^s$ —claims to the aggregate capital stock—to maximize lifetime utility. The value function of the individual of age  $s$  at period  $t$  is therefore:

$$V_t^s(a_{t-1}^{s-1}) = \max_{\{c_t^s, \ell_t^s, a_t^s\}} \{u(c_t^s, \ell_t^s) + \beta(1 - \gamma_t^s)V_{t+1}^{s+1}(a_t^s)\},$$

where  $\beta$  is the discount factor common to all individuals. Their age-specific mortality rate features in their total discount factor, with the terminal condition implied by the maximum lifespan  $T$  being  $\gamma_t^{T-1} = 1$ .

Individuals face an inequality constraint on the amount of labor they can supply at each point in time, with the upper bound being their normalized endowment of one unit of labor:  $\ell_t^s \leq 1$ . The participation rate of cohort  $s$ 's participation rate is therefore  $\ell_t^s$ , which implies that the labor force participation rate is an approximation to the extensive margin work-retire

decision as the individual gradually withdrawing from the labor market by reducing their number of working hours, as in [Krueger and Ludwig \(2007\)](#) and [Kulish et al. \(2006\)](#).

The unintentional bequests made by individuals of a household who die between periods are aggregated and redistributed as an annuity to the remaining living households of the same generation, following [Yaari \(1965\)](#) and [Blanchard \(1985\)](#). This assumption together with the age-specific mortality probability eliminates mortality risk.<sup>11</sup>

Individuals have age-specific productivities  $z^s$ , can receive a transfer from the government  $\xi_t^s$  (described below), receive a gross return  $R_t$  on last period's savings, and receives  $d_t^s$  for the redistributed unintentional bequest.<sup>12</sup> The period budget constraint of the individual is:

$$c_t^s + a_t^s \leq z^s w_t \ell_t^s (1 - \tau_t) + \xi_t^s + R_t a_{t-1}^{s-1} + d_t^s.$$

Consumption in the last period of life equals the return on an individual's remaining assets:

$$c_t^T \leq R_t a_{t-1}^{T-1}.$$

By assumption, the individual retires fully from the labor market in her last period of life.

### 3.2 Production

Firms hire labor and capital from individuals and operate a Cobb-Douglas production technology:  $y_t = k_{t-1}^\alpha \ell_t^{1-\alpha}$ . Aggregate capital is the sum of each cohorts's savings:  $k_t = \sum_s n_t^s a_t^s$ , while aggregate labor hired by the firm is in efficiency units of labor  $\ell_t = \sum_s z^s n_t^s \ell_t^s$ . Firms use the capital of the deceased in production.

The firm pays the marginal product of capital  $r_t = \alpha \frac{y_t}{k_{t-1}}$  and the marginal product of labor for the common component of wages:  $w_t = (1 - \alpha) \frac{y_t}{\ell_t}$ .

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<sup>11</sup>For an in-depth discussion of this point, see [Hansen and Imrohoroglu \(2008\)](#). The assumption of annuities markets in quantitative lifecycle models is common. See, for example, [Backus et al. \(2014\)](#). In the appendix, I consider the model where annuities are aggregated and distributed to all living agents. While this yields hump shaped consumption lifecycle profiles that better match observed profiles, the aggregate implications are largely similar to the model with annuities when the model's parameters are also recalibrated to ensure capital-output ratios are consistent with aggregate data.

<sup>12</sup>Due to the annuities markets, this can be also be written as scaling the return on savings  $a_{t-1}^{s-1}$  by  $1/(1 - \gamma_{t-1}^{s-1})$  (see [Hansen and Imrohoroglu, 2008](#)).

### 3.3 Government

The government taxes labor income at the rate  $\tau_t$  to fund a pay-as-you-go social security system. I include social security to reduce the pressure on individual savings required to fund consumption when retired, which helps to match capital-output ratios in the data while keeping other model parameters in a reasonable range. I follow [Attanasio et al. \(2007\)](#) in specifying that the benefit paid each period above an eligibility age  $T^*$  depends on three elements: the expected remaining life of the recipient, the accumulated pre-tax labor income of the worker, and a parameter  $\lambda$  governing the replacement rate of past earnings. Denote by  $W_t^s$  accumulated gross lifetime earnings, defined recursively as:

$$W_t^s = \begin{cases} w_t z^s \ell_t^s + W_{t-1}^{s-1}, & \text{if } s < T^* \\ W_{t-1}^{s-1}, & \text{if } s \geq T^*. \end{cases}$$

The amount  $\xi_t^s$  redistributed to an agent of age  $s \geq T^*$  depends on  $W_t^s$ :

$$\xi_t^s = \lambda \frac{W_t^s}{(T^* - 1)},$$

where the denominator reflects the amount of time that  $W_t^s$  is accumulated over. For those of age below the social security eligibility age  $T^*$ , the transfer  $\xi_t^s = 0$ . Households who are of an age above  $T^*$  can work for labor income and earn social security payments without penalty.<sup>13</sup> The government budget constraint is:

$$\sum_s n_t^s \xi_t^s = \sum_s n_t^s z^s w_t \ell_t^s \tau_t.$$

The tax rate  $\tau_t$  adjusts to equalize social security outlays and tax revenues.

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<sup>13</sup>I abstract from a reduction in retirement benefits resulting from the early taking of social security benefits. Such a model would require a choice of the part of the cohort between the ages of 62 and 67. I also abstract from a number of questions about the sustainability of pension systems in an aging society: see, for example, an in-depth analysis of these issues in [Attanasio et al. \(2007\)](#).

### 3.4 Equilibrium and solution concept

In the equilibrium, households and firms optimize on their choice of consumption, savings and labor. Each individual's optimal choice of consumption and savings delivers an Euler equation for consumption  $u_1(c_{t+1}^{s+1}, \ell_{t+1}^{s+1})/u_1(c_t^s, \ell_t^s) = \beta R_{t+1}$ . The optimal choice of labor reflects the marginal rate of substitution between consumption and leisure:  $u_1(c_t^s, \ell_t^s)/u_2(c_t^s, \ell_t^s) = z^s w_t$ . Firms set wages to the marginal product of labor:  $w_t = (1 - \alpha) \frac{y_t}{\ell_t}$ , and the gross return on capital to its marginal product  $r_t = \alpha \frac{y_t}{k_{t-1}}$ . Market clearing in the asset market requires that aggregate asset holdings equals capital demanded by firms:

$$k_t = \sum_s n_t^s a_t^s.$$

The labor market clears so that the total amount of labor supplied equates with the labor demanded by firms:

$$\ell_t = \sum_s z^s n_t^s \ell_t^s.$$

Goods market clearing pins down the real allocations of consumption and savings:

$$y_t = c_t + k_t - (1 - \delta)k_{t-1}.$$

Finally, the government budget constraint is balanced.

Computing the steady-state and transition between steady-states under exogenous shocks is not trivial and so a full characterization of the solution is left to the appendix.<sup>14</sup>

## 4 The demographic transition

This section discusses the calibration of the lifecycle model, the identification of the transitory and permanent demographic changes which enter the model, and its output and predictions under the demographic transition between 1940 and 2070.

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<sup>14</sup>In computing the path, I assume that there is perfect foresight over the evolution of the demographic variables. For a discussion, see [Attanasio et al. \(2007\)](#) and [Lucas \(2003\)](#).

## 4.1 Model calibration

A period of the model is calibrated to a year. Households are assumed to enter the model at 16 years of age and live for a maximum of 80 additional years, up to age 95.<sup>15</sup> I do not impose the full retirement of each generation until the last period of life, so that labor can be supplied up until age 95.<sup>16</sup>

There are two sets of demographic parameters which I calibrate to lifecycle data in the cross-section. First, I calibrate the age-productivity parameters  $z^s$  to the age-experience earnings profile. I follow [Elsby and Shapiro \(2012\)](#) in constructing the log experience-earnings profile. In constructing this profile, I use deflated data on full-time, full-year workers. The data is decennial Census data from 1960 to 2000, and annual American Community Survey data from 2001 to 2007.<sup>17</sup> In constructing the log experience-earnings profile, I pool high school dropouts, high school graduates, those with some college education, and those who have completed college or higher education.<sup>18</sup> Panel A of [Figure 2](#) plots the earnings-profile over age. The estimates imply a peak increase in wages of 85 log points, or about 134%, at age 45, before declining around the age of 50. This profile is in line with the estimate of [Guvenen et al. \(2015\)](#) who find an increase in the earnings of the mean worker of 127%.<sup>19</sup> Given the lack of reliable experience-earnings data on the productivity of older workers, after age 65, I calibrate the productivity of workers to decay by 20% a year.<sup>20</sup>

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<sup>15</sup>During the transition, the proportion of people who live past 95 is very small, being at most about 0.5% of the population.

<sup>16</sup>Given the low choice of labor supply at older ages, this choice is not too important. [Kulish et al. \(2006\)](#) show how *unanticipated* changes in life expectancy can change labor supply decisions deep into the period of retirement, as older workers with very few assets remaining return to the workforce to fund consumption during their unanticipated increase in the lifespan. In contrast, in my framework, all demographic changes are perfectly foreseen.

<sup>17</sup>Computed off IPUMS-USA extracts. A full description is given in the appendix.

<sup>18</sup>Clearly, a finer calibration would distinguish between workers of different education levels. In robustness exercises reported in the appendix, I distinguish between these education groups and analyze how unanticipated changes in the earnings profile map to labor supply decisions. In particular, [Kong et al. \(2016\)](#) finds flattening experience-earnings curves in synthetic cohorts constructed off IPUMS-USA data. The key results are quite robust to incorporating these findings, as discussed below.

<sup>19</sup>This number corresponds to the mean worker, which I align to because matching the means is more suitable when studying asset accumulation. [Guvenen et al. \(2015\)](#) show that the median worker experiences a much smaller increase of about 35%.

<sup>20</sup>This assumption is not too strong, given the small number of workers who remain in the labor force beyond 65 years of age. In the appendix I consider a productivity profile which holds productivity, for workers above age 65, fixed at the last observation at age 65, as in [Kulish et al. \(2006\)](#). This arguably aligns more

The second set of cross-sectional data that I calibrate the model to are the survival rates of each generation during the 80 years they could possibly live in the model. These  $\gamma_t^s$  parameters were chosen to match actuarial probabilities over time as reported by the Social Security Administration.<sup>21</sup> Calibrating to these probabilities also matches changes in the life expectancy of each generation over time, conditional on an individual reaching 16 years of age. The values used are the cohort-specific survival rates computed for the cohort year of birth. These profiles include both observed survival rates of cohorts up to their current age, and extrapolated survival rates based on the Social Security Administrations’s forecasts of life expectancy. I assume in the computations that all changes to these actuarial probabilities are exogenous and perfectly foreseen.<sup>22</sup> For the initial  $\gamma_t^s$  profile, I use the survival probabilities reported for those born in 1900 onwards. For those cohorts born before 1900 but who are alive in 1940, I use extrapolated values of the survival probabilities.<sup>23</sup> Panel B of **Figure 2** plots the survival probabilities for cohorts born between 1900 and 2070.

The period utility function takes the form  $\frac{(c_{t+s}^s)^{1-\rho}}{1-\rho} - v^s \frac{(\ell_{t+s}^s)^{1+\psi}}{1+\psi}$ . I impose structure on the disutility of providing labor  $v^s$  and follow [Kulish et al. \(2006\)](#) who use the functional form of a scaled cumulative density function of a normal distribution.<sup>24</sup> Panel C of **Figure 2** plots  $v^s$  over age. The disutility of labor supply is increasing in  $s$ , which is motivated by studies which link the disutility of work to deteriorating health status.<sup>25</sup> There are four parameters governing  $v^s$ : (i) one that implies a baseline level of disutility from labor, (ii) one that scales the entire disutility function, (iii) one that scales the age at which the disutility from work is increasing the most, and (iv) one that scales the pace at which the disutility increases with the profiles derived by [Casanova \(2013\)](#), who finds more or less a discrete level shift in earnings for older workers.

<sup>21</sup>These probabilities were sourced from Table 7 from the Cohort Life Tables for the Social Security Area by Calendar Year, in Actuarial Study No. 120 by Felicitie C. Bell and Michael L. Miller, available at: <https://www.ssa.gov/oact/STATS/table4c6.html>. A full description is given in the appendix.

<sup>22</sup>[Kulish et al. \(2006\)](#) study, by contrast, unanticipated changes in life expectancy.

<sup>23</sup>Because the survival probabilities are quite low for those years, the results are robust to alternative specifications and, in any event, are not that important for the model outcomes beyond 1970, which is the period I am interested in.

<sup>24</sup>The formula is left to the appendix.

<sup>25</sup>[Kulish et al. \(2006\)](#) also choose to make the function time-varying with increases in life-expectancy, with the result that the disutility of labor from employment declines in the gap between age and life-expectancy (see also, [Bloom et al., 2011](#)). I keep it constant to ensure age-participation rates do not vary significantly over time.

over age. These parameters are chosen so that the participation rate by age broadly matches observed participation rates in 2000. When I discuss the results, I plot in [Figure 7](#), age-specific participation rates in the data against the model’s predictions during the transition path.

For the social security system, I set the replacement ratio of accumulated earnings  $\lambda$  to 46.7%, the same value that is used in [Attanasio et al. \(2007\)](#), which in turn was based on the cross-country study of social security systems by [Whitehouse \(2003\)](#).

The remaining parameters independent of age are set to values which imply steady-state capital-output ratios that align with those in the Bureau of Labor Studies’ Multifactor Productivity (BLS-MFP) program and with the ratios implied by the measures of output and the capital stock constructed by [Fernald \(2012\)](#).<sup>26</sup> I set capital to depreciate by  $\delta = 10\%$  a year. The annual discount factor  $\beta$  is set to 0.995, which is high but less than the value used in [Attanasio et al. \(2007\)](#). The capital share  $\alpha$  is set to  $1/3$ , the average of the capital share reported by [Fernald \(2012\)](#) over 1948 to 2015. The intertemporal elasticity of substitution  $\rho$  is set to 2.5, and the inverse Frisch elasticity of labor supply  $\psi$  is set to 2.5, the central estimate of [Reichling and Whalen \(2012\)](#). Both values are consistent with the range cited in [Auerbach and Kotlikoff \(1987\)](#). The set of these parameters imply in the steady-state a capital-output ratio in 2000 of about 2.7, which is the observed capital-output ratio in the BLS-MFP in 2000.

## 4.2 Incoming cohort-size shocks

To capture the demographic dynamics of the baby boom, I choose anticipated shocks to the size of the incoming cohort so that the change in the observed cohort share is the same as the change in the model cohort share.<sup>27</sup> This approach imperfectly captures changes in the population distribution due to, for example, immigration.<sup>28</sup> Panel C of [Figure 3](#), discussed

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<sup>26</sup>These capital-output ratios varying between 2 and 2.7 over the period 1950 to 2013. A full description of these datasets is given in the appendix.

<sup>27</sup>Choosing initial population shocks to matching the *changes* is necessary because the implied steady-state cohort sizes under the  $\gamma_t^s$  profiles do not match the actual cohort sizes at each point in time, and the model is initialized at the 1940 steady-state. In practice, these choices are quantitatively minor and unimportant for the results.

<sup>28</sup>To understand how important migration is to changing the population structure, I considered statistics on the characteristics of immigrants, reported by The Migration Policy Institute, at [www.migrationpolicy.org](http://www.migrationpolicy.org).

later, plots this fraction over time, clearly showing the effect of the increase in fertility on the fraction of young workers in the economy. I assume that changes to the incoming population beyond 2015 decay to zero, so that the population distribution converges to the steady-state implied by the mortality profile that is constant from 2070.

### 4.3 Steady-state

Before exploring the full dynamics and to build intuition on the transition dynamics under the demographic trends, in this section I report how changes in the demographic structure in the model affects key economic quantities. As a point of comparison, in the model, permanently higher fertility has no steady-state effects on growth or the real interest rate. Higher fertility simply scales aggregate labor supply and capital in the steady-state, leaving factor prices unchanged.

Changes in longevity can, however, affect steady-state quantities by changing the distribution of the population by age. [Table 3](#) documents what the calibration implies for the capital-output ratio, the consumption-output ratio, the employment-population ratio, and the real interest rate under different mortality profiles and assuming that the economy is at a steady-state under those profiles. As longevity increases with declining mortality rates, aggregate savings increases to fund longer expected retirements. As a result, the capital-output ratio increases, the investment-output ratio declines (so that the consumption-output ratio increases) and the real interest rate falls as the marginal product of capital declines. These forces are familiar in lifecycle models with demographic changes (see, for example, the discussion in [Carvalho et al., 2015](#)).

Turning to the employment-population ratio, as people expect to live longer, the ratio is roughly constant. There are two opposing forces on the employment-population ratio in the model. The first is a compositional effect reminiscent of the analysis in [Section 2](#): as longevity increases, there are simply more older workers. Because of lower productivity and higher disutility of providing labor at older ages, those workers have lower employment participation

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The median age of immigrants in the US has increased from the 1980s/1990s to 2010. In 2014, the median age of immigrants was 43 years, compared to the 36 years in the general population.

**Table 3:** Steady-state over longevity profiles

| Quantity         | $\gamma^s$ fixed profiles |       |       |       |       |
|------------------|---------------------------|-------|-------|-------|-------|
|                  | 1940                      | 1970  | 2000  | 2030  | 2070  |
| $k/y$            | 2.325                     | 2.371 | 2.413 | 2.451 | 2.490 |
| $c/y$            | 0.751                     | 0.746 | 0.742 | 0.738 | 0.733 |
| $n/y$            | 0.628                     | 0.629 | 0.627 | 0.625 | 0.623 |
| $1 + r - \delta$ | 1.038                     | 1.035 | 1.033 | 1.030 | 1.028 |

rates, so that the overall rate declines. Acting against this is the second force: an increase in wages. As retirees accumulate capital, the marginal return to labor increases, incentivizing more labor supply. In the appendix, I consider the solution with exogenous labor supply and find that the participation rate falls 10 percentage points when workers cannot adjust their labor supply in response to wage movements induced by demographics. The equilibrium responses to wages can generate large movements in expected labor supply by age.

#### 4.4 Transition dynamics

This section presents the main results on the lifecycle model's predictions under the demographic transition. I discuss the key results on trends in the real interest rate, aggregate growth rates and productivity growth, and in the employment-population ratio.

##### 4.4.1 Demographic variables

Before discussing the effect of demographics on these economic quantities, I present in [Figure 3](#) summary measures of demographics from the model. Panel A plots life expectancy when individuals enter the model, Panel B plots the median age of those in the model, and Panel C presents the fraction of the population in the model at the entering age of 16. Life expectancy conditional on surviving to age 16 during the transition increases from about 77 years in 1950 to about 85 years in 2020. This profile matches exactly that observed in the data, because it is computed off observed conditional survival probabilities. The median age of those 16 and above from the model tracks that in the data over the transition. The slight difference between the two profiles is due to the initial population distribution and other population

changes like immigration which are not fully captured in the model. In the model, the initial population distribution reflects the steady-state associated with the mortality distribution of those born in 1940.

#### 4.4.2 Marginal product of capital and the real interest rate

The demographic trends imply a steadily declining path for the real interest rate in the model. The decline can be decomposed into a component due to increased longevity, and one due to changes in the composition of the workforce. This decomposition shows that the increase in longevity implies an approximately one percentage point decline in the real interest rate. This decrease is driven by the increase in savings demanded to fund consumption during longer expected retirement periods.

More notably, the aging of the baby boomer cohorts generates a large oscillation around the path implied by increasing longevity, falling from peak-to-trough between 1985 and 2015 by just over 2 percentage points. This pattern matches very closely over the same period, the decline in the real interest rate computed from the observed capital-output ratio provided by the Bureau of Labor Studies Multifactor Productivity Program.<sup>29</sup> The oscillation is driven by changes in the relative size of the workforce. The workforce is relatively young as the baby-boomers enter the labor market in the 1960s to 1980s, so that aggregate hours supplied is high relative to capital, thereby increasing the marginal return to capital. As the baby-boomer cohort ages and accumulate savings for retirement, the marginal return to capital and the real interest rate decline. This decline is then reinforced by the withdrawal of the baby-boomer cohort from the labor market, rapidly decreasing the marginal return to capital and staying low beyond 2030.

#### 4.4.3 Growth rates of aggregate quantities

What does the model say about how demographic trends affect the growth rates of aggregate quantities and productivity growth? Panel A of [Figure 5](#) presents the growth rates of total

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<sup>29</sup>In computing this observed series, I use the marginal product of capital and parameters of the model:  $1 + r - \delta = \alpha \frac{Y_t}{K_{t-1}}$ .

output, output per capita and output per worker. Total output growth due to demographics peaks at 1 percentage point just before 1980. The growth rate then steadily declines, until demographics becomes a drag on total output growth, which occurs in 2012. Demographic changes are then a substantial drag on total output growth, with the contribution to overall growth from demographics staying negative throughout the forecast horizon to 2070. In total, total output growth declines from peak-to-trough by just less than 2 percentage points.

Output per capita and per worker—productivity—growth rates show a very different pattern to the total growth rate. This difference is due to the entrance of the baby boomer generation into the workforce in the 1960s. Per worker and per capita output growth due to demographic changes is initially negative between 1960 and 1980, before becoming positive between 1980 and 2010. From then on, demographics causes per capita output growth to turn negative until at least 2040, while per worker output growth stays slightly negative over the forecast horizon. In total, per capita output growth declines from peak-to-trough by just over 1 percentage point between 1990 and 2025, while per worker output growth declines by about 0.7 percentage points over the same period.<sup>30</sup>

**Table 4** compares the model’s predictions for growth rates against those in the data. To make the model and data growth rates comparable, I add to the model’s growth rate an amount that ensures that the average growth rate from 1948 to 2015 is the same in the model and in the data. The model performs well in explaining the decline in average output growth across the 1990-1999 period and the 2010-2015 period, with the decline in average growth in the model being 0.71% compared to 0.86% in the data. The model generates about 30% of the decline in average capital growth and 40% of the decline in average consumption growth over the same period, but forecasts a steep decline in the growth rate of capital to the average levels observed between 2010-2015 by the mid-2020s.

Turning to per-person measures of output, the model explains almost all of the average decline in per capita output growth between 1990-1999 and 2010-2015, and explains 0.5 percentage points of the observed 1 percentage point decline in per hour output growth. Looking forward, the model forecasts a further decline in productivity of another 0.2 percentage

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<sup>30</sup>The magnitude of the decline in per capita growth accords with the results in [Antolin-Diaz et al. \(2014\)](#).

**Table 4:** Average annual growth rates

| Period                                            | Output |      | Capital |      | Consumption |      | $y/h$ |      | $y/n$ |      |
|---------------------------------------------------|--------|------|---------|------|-------------|------|-------|------|-------|------|
|                                                   | Model  | Data | Model   | Data | Model       | Data | Model | Data | Model | Data |
| <i>A. Model Against Raw Data Averages</i>         |        |      |         |      |             |      |       |      |       |      |
| 1990 to 1999                                      | 3.46   | 3.70 | 3.97    | 3.88 | 3.30        | 3.43 | 2.76  | 2.19 | 2.23  | 1.98 |
| 2000 to 2009                                      | 3.19   | 1.73 | 3.74    | 3.23 | 3.13        | 2.40 | 2.41  | 2.52 | 1.93  | 0.87 |
| 2010 to 2015                                      | 2.75   | 2.84 | 3.28    | 1.64 | 2.81        | 2.22 | 2.30  | 1.15 | 1.66  | 1.30 |
| <i>B. Model Against HP-filtered Data Averages</i> |        |      |         |      |             |      |       |      |       |      |
| 1990 to 1999                                      | 3.46   | 3.73 | 3.97    | 3.97 | 3.30        | 3.57 | 2.76  | 2.24 | 2.23  | 2.14 |
| 2000 to 2009                                      | 3.19   | 2.48 | 3.74    | 3.19 | 3.13        | 2.70 | 2.41  | 2.49 | 1.93  | 1.31 |
| 2010 to 2015                                      | 2.75   | 2.04 | 3.28    | 1.75 | 2.81        | 1.87 | 2.30  | 1.06 | 1.66  | 0.82 |
| <i>C. Model Forecasts</i>                         |        |      |         |      |             |      |       |      |       |      |
| Period                                            | Output |      | Capital |      | Consumption |      | $y/h$ |      | $y/n$ |      |
| 2016 to 2025                                      | 2.31   |      | 2.58    |      | 2.58        |      | 2.17  |      | 1.33  |      |
| 2026 to 2035                                      | 2.13   |      | 1.91    |      | 2.42        |      | 2.10  |      | 1.32  |      |
| 2036 to 2045                                      | 2.28   |      | 2.01    |      | 2.32        |      | 2.11  |      | 1.70  |      |
| 2046 to 2055                                      | 2.44   |      | 2.40    |      | 2.36        |      | 2.20  |      | 1.91  |      |
| 2056 to 2065                                      | 2.67   |      | 2.75    |      | 2.62        |      | 2.29  |      | 1.94  |      |

Notes:  $y/h$  is output per hour worked.  $y/n$  is output per person. To make the growth rates in the model and data comparable, I add to the model growth rate the amount that is needed to equalize the average growth rates in the model and data, computed over the years 1948 to 2015.

points, and 0.3 percentage points in per capita output growth, on average between 2026-2035.

What factors are driving these declines in average growth rates? In the model, there are three ways that these measures of output growth can change over time. Households can supply more hours of labor, affecting both output and the hours supplied. There are also changes in physical capital, as individuals save and consume out of accumulated savings in retirement. Consumption smoothing motives ensure that the level of savings changes at a different rate to the supply of labor. Third, the *quality* of labor can change. In particular, changes in the distribution of workers resulting from demographic changes alters the average skill-level of the workforce, which shows up in the productivity decomposition as fluctuations in the quality of labor.

Formally, I decompose the model's predictions for output growth and labor productivity growth into their component parts following a standard growth accounting exercise (Hall and Jones, 1999; Fernald, 2015). Consider the production function in the model:  $y_t = k_t^\alpha \ell_t^{1-\alpha}$

where  $\ell_t$  is aggregate efficiency units of labor. The total derivative of the production function decomposes the change in output into:  $\frac{dy_t}{y_t} = \alpha \frac{dk_t}{k_t} + (1 - \alpha) \frac{d\ell_t}{\ell_t}$ . In the lifecycle model, growth in output per efficiency unit of labor  $\ell_t$  arises from changes in aggregate labor supply or from changes in the labor quality of the workforce, as individual workers become more productive with age:  $\frac{d\ell_t}{\ell_t} = \frac{dh_t}{h_t} + \frac{dLQ_t}{LQ_t}$ , where  $h_t$  is aggregate hours and  $LQ_t$  is labor quality.

Panel B of [Figure 5](#) plots the decomposition for labor productivity growth, while Panel C plots the decomposition for total growth. Accelerating capital accumulation increases the growth rate of both labor productivity and total output up to 1995, after which the growth rate starts to decline. The number of hours provided has a large effect on productivity growth, but a positive effect on total growth, when the baby boomer cohorts enter the labor force around 1960.

A key component of both labor productivity and total growth is the change in the quality of the workforce which arises as the composition of the workforce interacts with the age-productivity profile. The decomposition implies that the contribution of the change in average labor quality to the growth rate of output and output per worker peaks around 1990, adding roughly 0.35 percentage points to total growth and productivity growth. The contribution of labor quality turns negative in 2000 as the mass of workers reaches the peak of the age-productivity profile, exhausting the potential for further growth in average human capital across the workforce. This force remains a drag on productivity growth until 2030.

Based on the model's predictions for how growth rates change with demographics, [Figure 6](#) plots the indexes of output, investment, and consumption in the data against their model counterparts, normalized to their 1990 values. I use the same procedure as for [Table 4](#) in adding the trend growth to the model series, and compare the resulting indexes to an index computed of each series' 1992 to 2007 pre-recession trend.<sup>31</sup> The trends that the model generates show clearly the decelerating profiles over time as compared to the rapid growth rates experienced over the 1992 to 2007 period. Comparing these trends to the data, the plots of the indexes show that the model does a good job in explaining the gap between the

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<sup>31</sup>This is the same time period that [Blanchard et al. \(2015\)](#) use in studying the slowdown in output growth from a pre-crisis path.

observed measures of output, investment and consumption relative to their pre-crisis trends.

A surprisingly strong feature of the data is the divergence between the levels of output per hour worked and output per capita. Demographic changes through the model can generate a gap between these variables because of different profiles for aggregate capital and aggregate hours. As workers withdraw from the labor market when they reach retirement age, their relative supply of capital to firms remains high, keeping total output high. As the workforce contracts, measured output per worker increases relative to the same amount of output per person alive. Panel D of [Figure 6](#) plots the ratio of total hours to the population, normalized to 1990, and shows that the model's predictions match the post-1990 trend in the data well.

#### 4.4.4 Employment-population ratio

I next turn to the predictions of the model for the employment-population ratio. [Figure 7](#) plots the share of the labor endowment that a member of each cohort chooses over time against participation rates observed in decennial censuses from 1960 to 2000, and annual American Community Surveys thereafter. A combination of the age-productivity profile and the calibration of the disutility of labor parameter at each age implies labor force participation rates that are humped shaped over age. The model is not parameterized to match the trend increase in female labor force participation over the years 1950 to 1990, accounting for its inability to match the substantial increase from 1950 to 1990.<sup>32</sup> The fluctuations in labor supply at each age are driven by equilibrium wages. These changes generate changes in labor force participation rates for older workers that match the observed changes.

[Figure 8](#) plots the aggregate employment-population ratio in the model against the data in Panel A, showing that under the calibration which was chosen to generate age-specific labor force participation rates that are consistent with those observed, the labor force participation rate declines in the model at the pace that is roughly as observed, and is predicted to continue to fall by about five percentage points from 2015 to 2030. This result is consistent with the results in [Section 2](#) and other reduced-form studies that assign much of the recent decline to

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<sup>32</sup>After 1990, the female labor force participation rate was roughly constant, and has been trending down since 2000.

demographic factors and, in particular, the age-compositional change in the population.

#### 4.5 Robustness and alternative calibrations

I do a number of experiments to verify that the baseline results are robust to changes in the calibration. The full results of each experiment are presented in the appendix but discussed briefly here. The first check is to see that the model's predictions hold when individuals face a constraint restricting their borrowing early in life. With borrowing constraints, the extent of savings is inflated, pushing up the capital-output ratio. As a result, the real interest rate is lower under this robustness check than the baseline model. The magnitude of the fluctuations of the real interest rate, the participation rate and output growth are very similar to the baseline model.

The remainder of the robustness checks focus on differences in the age-productivity profile. The second robustness check is to incorporate time-varying productivity profiles. The evidence presented in [Kong et al. \(2016\)](#) suggests that the age-productivity profiles have flattened over time. Such a flattening can affect the accumulation of human capital and can affect aggregate productivity measures in two ways: first, with a growth effect by lowering the potential for new workers to accumulate human capital, and second, with a level effect by affecting the productivity level that individuals enter the workforce on. I calibrate the age-productivity profiles by recomputing for each cross-sectional sample, the profile and then interpolating between those points in time. The overall effect on aggregate labor productivity is much the same as the baseline model as the two countervailing effects offset each other.

From 1985 on, the baseline predictions for the participation rate, aggregate labor productivity growth and the real interest rate are largely unaffected when the age-productivity and labor disutility profiles are calibrated to match female age-earnings profiles and female labor force participation rates from the 1940s to 1990s, after which female labor force participation is roughly constant. As a final point of comparison, I check that the aggregate predictions are robust to a calibration where an additional source of heterogeneity is modeled—where there are two types of workers, low or high skilled.

## 5 Demographics in the real business cycle model

The results of [Section 4](#) ascribe a significant role to demographics in explaining the trends observed in growth rates, employment-population ratios and the real interest rate. These results raise the question of the extent that demographic trends change how transitory shocks propagate to the rest of the economy ([Wong, 2015](#); [Jaimovich and Siu, 2009](#)). In my framework, I consider a question that has not been answered in the literature: to what extent is the zero lower bound more likely to be a binding constraint as the natural interest rate, and therefore the equilibrium nominal interest rate, trends down over the demographic transition? This analysis gets to a key issue in the secular stagnation debate: the extent to which post-crisis growth and the binding zero lower bound can be explained by secular trends against business cycle fluctuations.

To answer this, I first show in this section how heterogeneity in the lifecycle model at a point in time can be summarized in the real business cycle (RBC) model of [Kydland and Prescott \(1982\)](#) with the following four aggregate shocks: (i) as a productivity shock attached to aggregate output, (ii) as a labor-input shock, multiplying aggregate labor hours supplied, (iii) as shocks to the marginal utility of aggregate consumption, and (iv) as shocks to the disutility of aggregate effective hours supplied. The productivity shocks (i) and (ii) are aggregate supply shocks, while the shock (iii) to the marginal utility of consumption carries the interpretation of an aggregate demand shock. The shock to the marginal disutility of providing labor affects preferences and so affects the incentive to provide labor.

During a perfectly foreseen demographic transition, fertility shocks will change these four aggregate shocks in a way that is anticipated by agents in the model. I approximate the consequences of changes to mortality rates with a slight trend in the discount factor linked to the average mortality rate, and take the change in proportional taxes from the lifecycle model as given. In [Figure 10](#), I show that these paths approximate the lifecycle model well with the linearized approximation that I use.<sup>33</sup>

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<sup>33</sup>Discussed in [Section 6](#).

## 5.1 Heterogeneity as aggregate shocks

A full derivation of the aggregate shocks that describe the lifecycle model at a point in time is given in the appendix. For the derivation, I follow [Constantinides \(1982\)](#) and [Maliar and Maliar \(2003\)](#) and proceed in two steps. In the first step, I consider the problem of a social planner who maximizes a weighted sum of each individual's utility function. In the solution to this problem, the planner distributes aggregate consumption and aggregate labor supply between individuals alive in each period. In the second step, I solve the problem where the planner then maximizes these aggregate quantities subject to the economy's resource constraint. The solution to the planner's problem shows that there exists welfare weights on each individuals' utility function that equates the decentralized solution with the solution of the social planner.

The social planner's period utility function will be the preferences of the representative agent I am interested in. The solution in the appendix shows that this representative agent has preferences over aggregate consumption  $c_t$  and aggregate efficiency units of labor  $\ell_t$  with the period utility function taking the form:

$$U(c_t, \ell_t) = \phi_t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{v_t}{\phi_t} \left( \frac{\ell_t^{1+\varphi}}{1+\varphi} \right) \right].$$

The representative agent's problem will be to maximize this utility function over time with discount factor  $\beta$  by choosing aggregate consumption  $c_t$ , efficiency units of labor  $\ell_t$  and capital  $k_t$  subject to the economy's resource constraint and its production function  $y_t = \theta_t k_t^\alpha \ell_t^{1-\alpha}$ . As compared to the core real business cycle model in [Kydlan and Prescott \(1982\)](#), the solution gives the usual neoclassical growth model equations augmented with the shocks  $\phi_t$ ,  $A_t$ ,  $v_t$ , and  $\theta_t$ . Solving this problem gives an aggregate Euler equation:  $\lambda_t = \lambda_{t+1} R_{t+1} \beta$ , and the optimal conditions with respect to aggregate consumption:  $\phi_t c_t^{-\sigma} = \lambda_t$ , and aggregate labor:  $v_t \ell_t^\varphi = w_t \lambda_t$ . The resource constraint binds:  $y_t = c_t + k_t - (1 - \delta)k_{t-1}$ , and the marginal products of labor and capital are:  $w_t = (1 - \alpha)\theta_t \left( \frac{k_{t-1}}{\ell_t} \right)^\alpha$ , while the gross return on capital is:  $R_t = \alpha\theta_t \left( \frac{\ell_t}{k_{t-1}} \right)^{1-\alpha} + 1 - \delta$ . The relationship between the efficiency units of labor and

aggregate hours is  $\ell_t = A_t h_t$ .

These four shocks in the solution show how the demographic state affects the aggregate economy. First, in the way that demographics affects aggregate productivity  $\theta_t$ . The second is the process  $A_t$  that attaches to the labor input  $\ell_t$  and the aggregate supply of hours  $n_t$ . The third is the way that demographics affects the economy's marginal utility of consumption  $\phi_t$  affecting aggregate demand. The fourth is how demographics influences the aggregate disutility of providing labor  $v_t$ , which, together with the preference shock  $\phi_t$ , attaches to the labor wedge.

An important assumption that is needed for the derivation is the availability of annuities on the unintentional bequests of those who die between periods. Because of the annuity markets, the individual faces no uncertainty along the demographic path. With claims to capital, they can perfectly smooth consumption along that path. The result of this complete markets setup is that the ratio of marginal utilities of wealth across all individuals is the same at any point in time, which implies the existence of the welfare weights that attach to each individual's utility function in the planner's problem. Those weights equate the planner's solution with the decentralized equilibrium.

Changing demographics will, over time, affect the values of these aggregate shocks. The paths of these aggregate shocks are given in Figure 9. I discuss each shock in turn.

## 5.2 Productivity shocks

The first shocks the derivation gives are productivity shocks shifting total output  $\theta_t$  and shifting labor-input  $A_t$ . Substituting into the production function the definition of efficiency units of labor  $\ell_t = A_t h_t$  gives the production function:

$$y_t = (\theta_t)^{1-\alpha} k_t^\alpha (A_t h_t)^{1-\alpha}.$$

In the appendix, I show that  $\theta_t$  and  $A_t$  are:

$$\theta_t = \sum_s n_t^s z^s, \quad \text{and} \quad A_t = \frac{\sum_s n_t^s (\hat{z}^s)^{1+1/\varphi} (v^s \lambda^s)^{-1/\varphi}}{\sum_s n_t^s (\hat{z}^s)^{1/\varphi} (v^s \lambda^s)^{-1/\varphi}},$$

where the value  $\hat{z}^s = z^s/\theta_t$  denotes the individual  $s$ 's skill level relative to the average skill level in the economy  $\theta_t$ , and the  $\lambda^s$  parameters are the Pareto weights attached to an individual of age  $s$ . The shock  $\theta_t$  reflects the increase in productivity caused by changes in the size and composition of the workforce over idiosyncratic skill levels.  $A_t$  has the interpretation as a population-weighted average of individual labor and skills supplied, reflecting the amount of hours needed to obtain an effective unit of labor. It incorporates both relative productivities and the disutility of providing labor. In principle,  $\theta_t$  and  $A_t$  are straightforward to compute, requiring only hours by age and assumptions about the age-productivity profile, the age-disutility of work profile, Pareto weights that attach to each generation, and a value for the inverse Frisch elasticity of labor supply  $\varphi$ .

These two shocks summarize how productivity moves during the demographic transition. They are plotted in Panel A and Panel B of [Figure 9](#). There is both a level effect and a growth effect. To build intuition, consider the value of the wedge absent the population shocks to the 16 year-old cohorts. Owing to the larger size of the population which occurs as individuals live for longer, the shock to  $\theta_t$  gradually increases over time. The labor-input shock, however, gradually decreases over time absent population shocks.

Over the full demographic transition, the aggregate productivity shock increases with the size of the population, which swells as the baby boomer generations enter the workforce. The increase in aggregate productivity peaks in the year 2015, after which it declines as workers exit the labor market.

### 5.3 Preference shock

The second shock is a preference shock attached the marginal utility of consumption:

$$\phi_t c_t^{-\sigma} = \lambda_t.$$

In the appendix, I show that this shock has a simple expression:

$$\phi_t = \left[ \sum_s n_t^s \lambda^s \right]^\sigma.$$

The aggregate preference shock simply maps to the size of the population at each point in time, and is increasing in the curvature of the utility function  $\sigma$ . It can be conceptualized as an aggregate demand shock, incentivizing increased consumption.

To understand how demographics affects this shock, take first the profile without initial population shocks. There are anticipated increases in life expectancy experienced by the young in a period, represented by a steady decline in the mortality parameters  $\gamma_t^s$ . Owing to the increased number of people in the economy, this implies a steady increase in the size of the population and therefore in the aggregate demand factor. The marginal utility of consumption is inflated, reflecting more consumption at the aggregate level.

Adding fertility shocks generates a large increase in aggregate demand starting in the 1960s and peaking around 2020. This reflects the increased size of the population during the transition. The strength of the demographic transition on demand reflects consumption smoothing. As the utility function becomes more linear in consumption, the effect of demographics on the demand shock decreases. As the desire to smooth consumption diminishes, there is less transitory changes to the value of aggregate consumption at a point in time.

Over the full demographic transition, the demand shock peaks around 2020, which is *after* the peak in the productivity shock. This reflects consumption smoothing on the part of households, with their consumption staying high after they exit the labor force.<sup>34</sup> The preference shock is at an elevated level around the period of the Great Recession. This suggests that a given percentage change in the aggregate discount factor scales with the value of the aggregate demand shock, providing a mechanism to justify a large preference shock around the time of the recession, consistent with a number of papers that use large discount factor shocks to model the Great Recession and ensure the nominal interest rate hits the zero lower bound (see, for example, [Eggertsson and Woodford, 2003](#)).

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<sup>34</sup>This observation suggests implications for asset prices more generally. In particular, the consumption utility shock at this point is less pronounced than the labor productivity shock, suggesting strong demand for an asset like housing in the run-up to the financial crisis.

## 5.4 Shock to the marginal disutility of labor

The third shock,  $v_t$ , is an aggregate shock to the marginal disutility of labor which attaches to the labor wedge. Equating the marginal utility of consumption and the marginal disutility of labor, and substituting in for hours worked gives:

$$\frac{w_t}{\ell_t^\varphi / c_t^{-\sigma}} = \frac{v_t}{\phi_t}.$$

This says that a component of the measured labor wedge is the ratio of the two demographic shocks  $v_t/\phi_t$ , where  $v_t$  is a population-weighted average of individual disutilities of providing labor, shown in the appendix to be:

$$v_t = \left[ \sum_s n_t^s (\hat{z}^s)^{\frac{1}{\psi} + 1} (v^s)^{-\frac{1}{\psi}} \right]^{-\psi}.$$

An increase in the average disutility of labor and the average productivity of labor increases the labor wedge. Higher consumption preference shocks and lower labor taxes reduce the wedge. As compared to the consumption preference and productivity shocks, the new component here is the aggregate disutility of providing labor,  $v_t$ . The trends from demographics affect the wedge from both deviations

Over the demographic transition, the value of  $v_t$  increases substantially to peak in the year 2000, before decreasing. Considering the path without fertility shocks and inspecting the expression for  $v_t$  from the derivation reveals why. There are two lifecycle aspects to  $v_t$  that are relevant. The first is labor productivity by age. The second component is the disutility by age. With no fertility shocks,  $v_t$  trends up, as higher population weight is placed on higher disutilities by age so that, on average, there is a higher disutility of supplying labor.

## 5.5 Approximation for longevity shocks and proportional taxes

The final anticipated shock that is needed to ensure the business cycle model gives the same predictions as the lifecycle model is a shock to the aggregate discount factor to account for

variation in the average mortality rate over time.

In the lifecycle model, the pay-as-you-go social security system is financed by proportional taxes to labor income. The tax rate varies to ensure the government budget is balanced each period. I approximate the way that this shock changes the wage by taking the path of labor income taxes as given and fully anticipated from the lifecycle model.

To verify that these shocks recover the paths of aggregate variables from the global solution to the lifecycle model, I take the aggregate series implied by the lifecycle model and compare it to the linear approximation that is computed under anticipated shocks, under the assumption that the Pareto weights attaching to each individual are the same. The solution method is described in more detail in the next section. The two series are plotted in [Figure 10](#) and verifies that the aggregate representation generates paths for aggregate variables which are very close to those generated by the full solution of the lifecycle model.

## **6 Aggregate shocks along the demographic transition path**

In this section, I discuss the method used to solve the aggregate model subject to anticipated and permanent changes to the shocks uncovered in the previous section. I then use the solution method to estimate a New Keynesian model which incorporates these anticipated demographic shocks. With the estimated model, I study how the inclusion of demographic trends affects the interpretation of how transitory shocks drive business cycle movements, with a focus on changes in volatility induced by demographic structural changes and on the likelihood that the zero lower bound binds during the demographic transition.

### **6.1 Solution method**

The standard linear, time-invariant solution cannot be used in approximating the model for two reasons: first, because the demographic shocks are changes to the wedges attaching to the equations of the model and, additionally, are fully anticipated by agents in the economy, and second, because the zero lower bound binds for a large part of the 1984Q1 to 2015Q1 sample I use. In this case, I adapt a methodology based on anticipated structural changes to the

parameters of the model, where the sequence of demographic shocks is taken as an anticipated path of the structural parameters of the model. The final structure of the economy is the one that arises at the completion of the demographic transition—under my calibration, this final structure applies from the year 2070 onwards. Conceptualizing the demographic shocks derived in [Section 5](#) as anticipated changes to the model’s structural parameters shows how demographics can change the transmission of transitory shocks, as they change the underlying structure of the economy.<sup>35</sup>

To describe the full time-varying approximation, I first consider the time-invariant approximation of a rational-expectations model of the form  $x_t = \Psi(x_{t-1}, \mathbb{E}_t x_{t+1}, \varepsilon_t)$  where  $x_t$  is the vector of model variables (state and jump), and  $\varepsilon_t$  is a vector of exogenous unanticipated shocks whose stochastic properties are known. The rational-expectations approximation of the model, linearized around its steady-state, is written as:

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}\varepsilon_t, \quad (3)$$

where  $x_t$  is of size  $n \times 1$  and  $\varepsilon_t$  is of size  $l \times 1$ . A solution to (3), following [Binder and Pesaran \(1995\)](#), is written as:

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t.$$

where  $\mathbf{J}$ ,  $\mathbf{Q}$  and  $\mathbf{G}$  are conformable matrices which are functions of the matrices  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ ,  $\mathbf{D}$  and  $\mathbf{F}$  which contain the structural parameters of the model.<sup>36</sup>

Now consider the case where model agents have time-varying beliefs about the evolution of the model’s structural parameters:  $x_t = \Psi_t(x_{t-1}, \mathbb{E}_t x_{t+1}, \varepsilon_t)$ . Denote the corresponding structural matrices for the model linearized at each point in time around the steady-state

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<sup>35</sup>In particular, see [Canova et al. \(2015\)](#), [Kulish and Pagan \(2012\)](#), and [Fernández-Villaverde et al. \(2007\)](#). More generally, [Jones \(2015a\)](#) discusses how the zero lower bound is a change in the structural parameters of the monetary policy rule that applies for a state-contingent period, or a period that is governed by a forward guidance motive. As is recognized from the work of [Christiano et al. \(2015\)](#) on government spending multipliers, the interpretation of transitory structural shocks can differ when the zero lower bound binds.

<sup>36</sup>See the appendix for a description of the full time-invariant solution.

corresponding to the time  $t$  structural parameters are  $\mathbf{A}_t$ ,  $\mathbf{B}_t$ ,  $\mathbf{C}_t$ ,  $\mathbf{D}_t$  and  $\mathbf{F}_t$ .<sup>37</sup> A solution to the problem with time-varying structural matrices exists if agents in the model expect the structural matrices to be fixed at a future point in time at values which are consistent with a time-invariant equilibrium (Kulish and Pagan, 2012).<sup>38</sup> In this case, the solution has a time-varying VAR representation:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t, \quad (4)$$

where  $\mathbf{J}_t$ ,  $\mathbf{Q}_t$  and  $\mathbf{G}_t$  are conformable matrices which are functions of the evolution of beliefs about the time-varying structural matrices  $\mathbf{A}_t$ ,  $\mathbf{B}_t$ ,  $\mathbf{C}_t$ ,  $\mathbf{D}_t$  and  $\mathbf{F}_t$ . The appendix derives in full the time-varying solution.<sup>39</sup>

## 6.2 Adding frictions and monetary policy

To the real business cycle core derived from the lifecycle model, I add quadratic capital adjustment costs, Rotemberg pricing frictions and a monetary policy rule responding to inflation and the level of output relative to its steady-state value. The nominal interest rate is additionally subject to the zero lower bound. In the steady-state, the model reduces to the steady-state of the core real business cycle model derived from the lifecycle framework.<sup>40</sup>

## 6.3 Estimation with demographic trends and the zero lower bound

The solution (4) can be used to set up the state-space representation and likelihood methods used to estimate the model. In estimating the model, I include the zero lower bound period, 2009Q1 to the end of the sample 2015Q1, by incorporating as data series summarizing how

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<sup>37</sup>One can linearize the model around its original steady-state or the steady-state associated with the time-varying system's final structure. Given the somewhat large movements in the steady-state induced by demographic changes, I chose the former approach, linearizing each set of structural matrices around the steady-state implied by that structure.

<sup>38</sup>Also see Jones (2015a) and Guerrieri and Iacoviello (2015), who apply this procedure to approximating models with occasionally binding constraints quickly and efficiently.

<sup>39</sup>The methodology is very general and applicable to many other problems of interest. The zero lower bound, forward guidance and changes to the steady-state level of inflation and output growth are anticipated changes to the model's structural parameters which can be handled by solution (4) (Jones, 2015a).

<sup>40</sup>A full description of the additional frictions and monetary policy is given in the appendix.

long the zero lower bound is expected to be a binding constraint at each period it is observed to bind. Expliciting connecting these series to an observable quantity allows for the possibility of forward guidance as in [Kulish et al. \(2014\)](#) and [Jones et al. \(2016\)](#). I include the zero lower bound period and machinery to better identify the parameters of the monetary policy rule and the shock processes. I employ a Bayesian estimation using priors common to the literature and evaluate the convergence of the estimated posterior distributions in the usual way. The full estimation results are discussed at length in the appendix.

#### **6.4 Simulations of the economy during the demographic transition**

I examine the interaction between demographic trends in affecting the volatility of output and the importance of the zero lower bound by simulating the estimating New Keynesian model many times and studying the distribution of paths of output and the nominal interest rate over those simulations.

To what extent have demographic changes made the zero lower bound a binding constraint?

[Figure 11.](#)

#### **6.5 Shocks over the Great Recession [TBD]**

In this section, I extract the structural business cycle shocks and compare the observed path to a counterfactual where demographic variables are held at their 1984 levels. This allows me to ask the extent to which demographics has constrained the nominal interest rate since the Great Recession.

### **7 Conclusion**

This paper set out to achieve two goals: first, to understand the consequences of the aging of the US population for the trends in real interest rates, output and productivity growth, and for the employment-population ratio—variables whose trends underpin and motivate the secular stagnation hypothesis. I used a lifecycle model with overlapping generations to show that declining mortality rates and changes to the population share of the young can generate

trends that are close to those observed. The model assigns a large part of the explanation for these trends to these demographic trends.

The second goal of the paper was to analyze the business cycle implications of an aging population. I examined the question: how do demographic trends affect the likelihood that the zero lower bound will be a binding constraint? To answer this, I develop an approximate aggregate representation of the lifecycle economy and add monetary policy and pricing frictions. I then estimate the model using Bayesian techniques over a period including the zero lower bound period and under the demographic transition. With the estimated model, I study stochastic simulations and find strong evidence of time-dependence in the probability that the zero lower bound will bind, with about 40% of the density of simulated paths of the nominal interest rate lying below the zero lower bound between 2010 and 2020.

Perhaps the most consequential assumption that is made in the lifecycle model is that the age-productivity profile is held constant over time and over the demographic transition. [Maestas et al. \(2016\)](#) find an interesting correlation between productivity growth and the age-composition of the workforce. My model is silent on the extent to which demographic changes might have a feedback effect on the slope of the age-productivity profile; an interesting extension would be to explore those feedback effects, perhaps in an endogenous growth framework where workers learn their productivities from others ([Lucas and Moll, 2011](#)).

The most interesting extension of this framework would be to incorporate housing and to examine whether demographics can justify the housing price dynamics observed in the run up to the 2008 financial crisis. The results presented motivate this extension, as I find that the consumption demand and labor productivity shocks derived for the aggregate representation peak in different periods, with consumption demand peaking around 2020, roughly 10 years after the peak in the aggregate productivity shock.

The business cycle framework incorporating demographic changes opens up a number of promising avenues for future research. To what extent do demographic trends explain the Great Moderation? How does the demographic state affect the size of the response to transitory shocks? These are questions that can be answered readily using the methodology

outlined in this paper.

# Appendix

## A Lifecycle asset and consumption profiles

The lifecycle profiles for consumption and savings in the initial steady state in the year 1940 are given in [Figure A.1](#). Individuals initially borrow. Their borrowing peaks around the age of 60. This is consistent with the Survey of Consumer Finances median assets from the SCF combined extract. Consumption rises over the lifecycle with slope  $\beta R$ . This is a consequence of full annuities and the Euler equation.

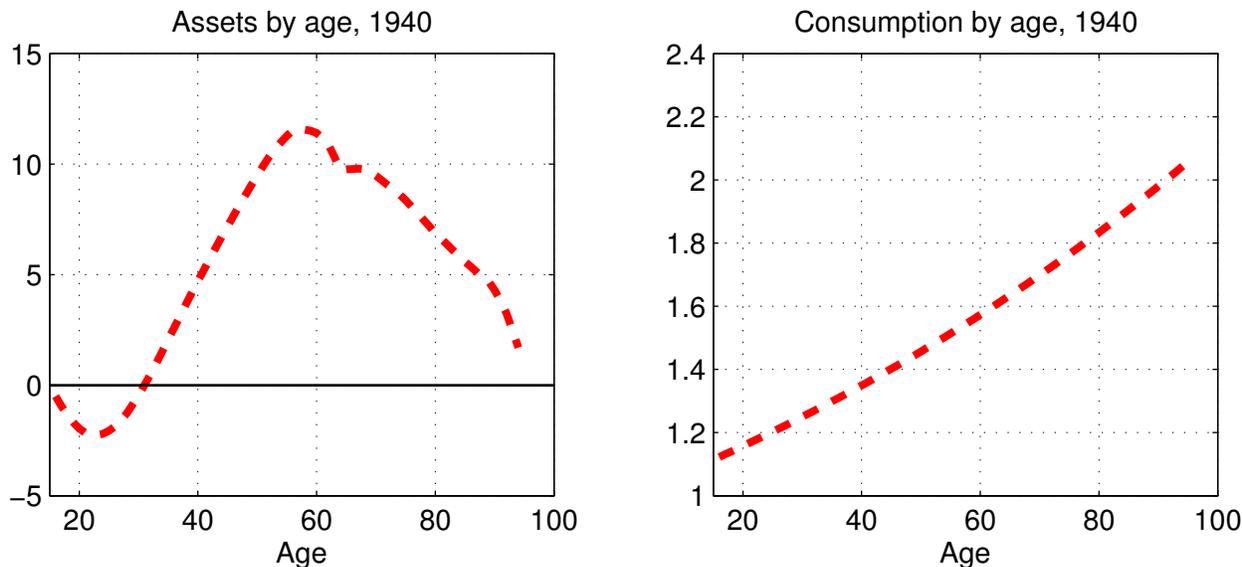


Figure A.1: Lifecycle profiles for consumption and savings.

## B Algorithm to solve path of OLG model

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## C Lifecycle model robustness

### C.1 Disutility of labor calibration

As in [Kulish et al. \(2006\)](#), the equation for the disutility of labor supply is:

$$v^s = \kappa_0 + \left(\kappa_1 \frac{s}{70}\right) \int_{-\infty}^s \frac{1}{70\sqrt{2\pi}\kappa_3} \exp\left(-\frac{1}{2} \left[\frac{x - 70\kappa_2}{70\kappa_3}\right]^2\right) dx.$$

The function is a scaled version of the cumulative density function of a normal distribution.  $\kappa_0$  vertically shifts the function.  $\kappa_1$  scales the increasing part of the function.  $\kappa_2$  shifts the mean of the distribution, or the age at which disutility starts to increase.  $\kappa_3$  shifts the standard deviation of the function, or the slope for which the disutility increases.

## C.2 Borrowing constraints

In this robustness exercise, I set  $a_t^s \geq 0$  for all  $t, s$ . The effect of this change is to cause the young to supply less labor for the periods when borrowing is constrained. The consumption profile is steep for those periods the young are constrained. In the aggregate, with the other calibrated parameters kept constant, there is more aggregate savings and less aggregate labor supplied, resulting in a higher capital-output ratio and lower real interest rate. The movements in the interest rate and labor force participation rate are largely unaffected as compared to the baseline results.

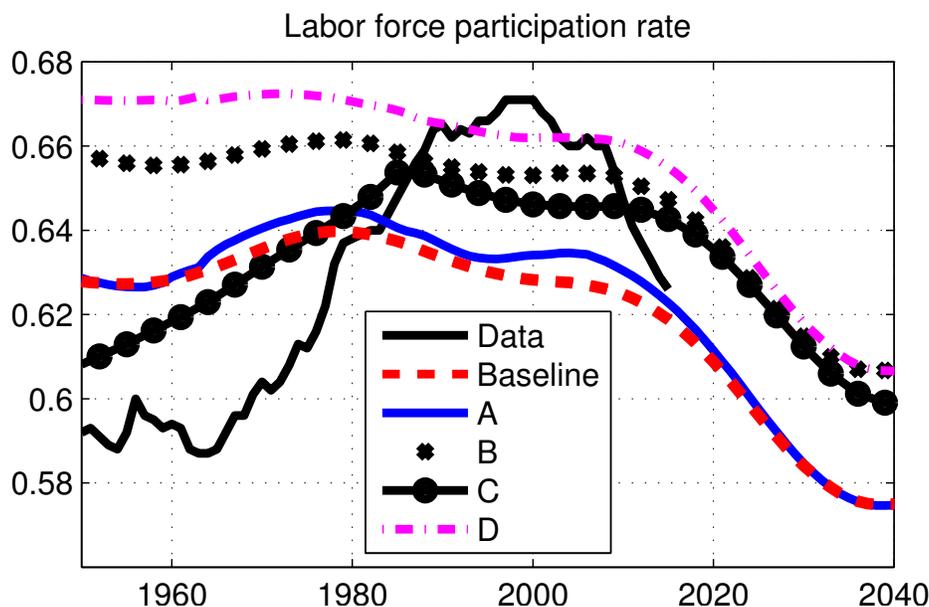


Figure C.1: Robustness.

## C.3 Exogenous labor supply

In this exercise, households have no disutility of supplying labor, and are forced to enter retirement full-time at age 65. This exercise changes substantially the profiles for aggregate labor force participation and for the real interest rate.

## C.4 Time-varying productivity profiles

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## C.5 Gender-based calibration

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## C.6 Skill-based calibration

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## D Heterogeneity in lifecycle model as aggregate shocks

The measure of agents of age  $s$  in period  $t$  is  $n_t^s$ .<sup>41</sup> Take an individual  $j$  belonging to the cohort born in period  $s$ . Rewrite her lifecycle problem as an infinite horizon problem with a preference process  $\phi_t^{j,s}$  that proxies for the lifecycle. Her preferences are therefore:

$$\max_{\{c_t^{j,s}, \ell_t^{j,s}, k_t^{j,s}\}} \sum_{\tau=s}^{\infty} \beta^\tau \left[ \prod_{r=s}^{\tau} (1 - \gamma_r^s) \right] \phi_\tau^{j,s} u(c_\tau^{j,s}, \ell_\tau^{j,s}). \quad (5)$$

The individual's sequence of budget constraints is as in the text. Let  $\lambda_t^{j,s}$  be the marginal utility of wealth, or the Lagrange multiplier on the individual's budget constraint. Because of the presence of full annuities markets for unintentional bequests, the individual's savings decision implies the Euler equation:

$$\lambda_t^{j,s} = \lambda_{t+1}^{j,s} R_{t+1} \beta. \quad (6)$$

Optimizing (5) by choices of consumption and labor yields the first order conditions, first for the marginal utility of consumption:  $\phi_t^{j,s} u_1(c_t^{j,s}, \ell_t^{j,s}) = \lambda_t^{j,s}$ , and second, for the labor supply decision:  $\phi_t^{j,s} u_2(c_t^{j,s}, \ell_t^{j,s}) = \lambda_t^{j,s} z_t^s w_t$ . The Euler equation (6) implies that for any two individuals  $j, i$  that the ratios of their marginal utilities is constant for all time periods  $t, t'$ , which implies:

$$\frac{\lambda_t^{j,s}}{\lambda_t^{i,s'}} = \frac{\lambda_{t'}^{j,s}}{\lambda_{t'}^{i,s'}} = \frac{\lambda^{i,s'}}{\lambda^{j,s}},$$

where  $\lambda^{j,s} = \frac{\lambda_t}{\lambda_t^{j,s}}$ . Using this condition, we can rewrite the first order conditions for the household's problem as:

$$\lambda^{j,s} \phi_t^{j,s} u_1(c_t^{j,s}, \ell_t^{j,s}) = \lambda_t, \quad (7)$$

and:

$$\lambda^{j,s} \phi_t^{j,s} u_2(c_t^{j,s}, \ell_t^{j,s}) = \lambda_t z_t^s w_t. \quad (8)$$

These two equations, together with each individual's budget constraints and aggregate definitions, characterize the decentralized economy.

Now consider a period-by-period problem of a social planner who chooses consumption and labor supply for each individual in each cohort to maximize the sum of individual utilities, weighted by welfare weights  $\lambda^{j,s}$ :

$$U(c_t, \ell_t) = \max_{\{c_t^{j,s}, \ell_t^{j,s}\}} \left\{ \sum_s \int \lambda^{j,s} \phi_t^{j,s} u(c_t^{j,s}, \ell_t^{j,s}) dj \right\}, \quad (9)$$

subject to the definitions for total consumption ( $c_t = \sum_s n_t^s c_t^{j,s}$ ) and for total labor supplied in efficiency unit terms ( $\ell_t = \sum_s n_t^s z_t^s \ell_t^{j,s}$ ). Because individuals within a cohort are identical and the measure of individuals within a cohort is given by  $n_t^s$ , we can write this problem as:

$$U(c_t, \ell_t) = \max_{\{c_t^{j,s}, \ell_t^{j,s}\}} \left\{ \sum_s n_t^s \lambda^{j,s} \phi_t^{j,s} u(c_t^{j,s}, \ell_t^{j,s}) \right\}.$$

<sup>41</sup>Note, this notation is slightly different from the text, where  $s$  refers to the birth date.

Letting the Lagrange multiplier on the definitions of total consumption and labor be  $\varphi_t$  and  $\nu_t$  respectively, the first order conditions of this static problem are:

$$n_t^s \lambda^{j,s} \phi_t^{j,s} u_1(c_t^{j,s}, \ell_t^{j,s}) = n_t^s \varphi_t,$$

and:

$$n_t^s \lambda^{j,s} \phi_t^{j,s} u_2(c_t^{j,s}, \ell_t^{j,s}) = n_t^s \nu_t z_t^s.$$

The envelope conditions for the problem (9) on  $c_t$  and  $\ell_t$  are simply  $\varphi_t$  and  $\nu_t$ , respectively, so that  $U_1(c_t, \ell_t) = \varphi_t$  and  $U_2(c_t, \ell_t) = \nu_t$ .

Now consider the problem where the planner optimizes by choice of total consumption, total efficiency units of labor supplied  $\ell_t$ , and total savings, the social utility function over time:

$$\max_{\{c_t, k_t, \ell_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t),$$

subject to the economy's resource constraint each period. Letting  $\lambda_t$  be the Lagrange multiplier on the resource constraint, the first order conditions of this problem imply the same expressions as those equations that characterize the decentralized economy's problem. In particular, we get for the choice of aggregate savings:  $\lambda_t = \lambda_{t+1} R_{t+1} \beta$ . For aggregate consumption, the first order condition implies the standard condition:  $U_1(c_t, \ell_t) = \lambda_t$ , and for the choice of labor,  $U_2(c_t, \ell_t) = \lambda_t w_t$ , where I have substituted in  $w_t$  for the marginal product of labor.

To derive expressions for the shocks, the particular setup of the individual's problem in the lifecycle model has  $u(c_t^{j,s}, \ell_t^{j,s}) = \frac{(c_t^{j,s})^{1-\sigma}}{1-\sigma} - v^s \frac{(\ell_t^{j,s})^{1+\varphi}}{1+\varphi}$ . The  $\phi_t^{j,s}$  are one when the individual is alive and zero otherwise. The skill of each individual  $z_t^s$  is positive in periods the individual is alive and zero otherwise. The first order conditions for the optimization of the social utility function are:

$$\lambda^{j,s} \phi_t^{j,s} (c_t^{j,s})^{-\sigma} = \varphi_t,$$

and:

$$\lambda^{j,s} \phi_t^{j,s} v^s (\ell_t^{j,s})^\varphi = \nu_t z_t^{j,s}.$$

This implies:

$$c_t^{j,s} = \varphi_t^{-1/\sigma} (\lambda^{j,s} \phi_t^{j,s})^{1/\sigma},$$

and:

$$z_t^{j,s} \ell_t^{j,s} = \nu_t^{1/\varphi} (z_t^{j,s})^{1+1/\varphi} (v^s \phi_t^{j,s} \lambda^{j,s})^{-1/\varphi}.$$

Integrating (summing) these expressions with respect to individuals (cohorts) gives aggregate consumption:

$$c_t = \sum_s n_t^s c_t^{j,s} = \varphi_t^{-1/\sigma} \left( \sum_s n_t^s (\lambda^{j,s} \phi_t^{j,s})^{1/\sigma} \right).$$

For aggregate labor supplied in efficiency units:

$$\ell_t = \sum_s n_t^s z_t^{j,s} \ell_t^{j,s} = \nu_t^{1/\varphi} \left( \sum_s n_t^s (z_t^{j,s})^{1+1/\varphi} (v^s \phi_t^{j,s} \lambda^{j,s})^{-1/\varphi} \right).$$

These expressions imply that individual consumption and labor supply are fractions of their respective aggregates:

$$c_t^{j,s} = \frac{(\lambda^{j,s} \phi_t^{j,s})^{1/\sigma}}{\sum_s n_t^s (\lambda^{j,s} \phi_t^{j,s})^{1/\sigma}} c_t, \quad \text{and} \quad \ell_t^{j,s} = \frac{(z_t^{j,s})^{1/\varphi} (v_t^s \phi_t^{j,s} \lambda^{j,s})^{-1/\varphi}}{\sum_s n_t^s (z_t^{j,s})^{1+1/\varphi} (v_t^s \phi_t^{j,s} \lambda^{j,s})^{-1/\varphi}} \ell_t.$$

More compactly,  $c_t^{j,s} = \chi_1^{j,s} c_t$  and  $\ell_t^{j,s} = \chi_2^{j,s} \ell_t$ .

Substituting these expressions into the social utility function gives:

$$U = \sum_s n_t^s \lambda^{j,s} \phi_t^{j,s} \left( (\chi_1^{j,s})^{1-\sigma} \frac{c_t^{1-\sigma}}{1-\sigma} - v_t^s (\chi_2^{j,s})^{1+\varphi} \frac{\ell_t^{1+\varphi}}{1+\varphi} \right),$$

which can be rearranged to get an aggregate utility function:

$$U = \phi_t \frac{c_t^{1-\sigma}}{1-\sigma} - v_t \frac{\ell_t^{1+\varphi}}{1+\varphi}.$$

In this representation, aggregate labor  $\ell_t$  is expressed as efficiency units of labor. Reorganizing this gives:

$$U = \phi_t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{v_t}{\phi_t} \left( \frac{\ell_t^{1+\varphi}}{1+\varphi} \right) \right].$$

The process  $\phi_t$  can be interpreted as a usual preference shock while  $v_t/\phi_t$  is the labor wedge on the efficiency units of labor supplied. Finally, if we know the welfare weights  $\{\lambda^{j,s}\}_{j,s}$ , all terms in  $\phi_t$  and  $v_t$  are exogenous and can be computed.

The final expression to determine is the term which converts aggregate supply of units of labor, denoted by  $\ell_t$ , into  $\ell_t$ , the aggregate supply of efficiency units of labor which enters the firm's production function. To get this shock, we integrate  $\ell_t^{j,s}$  over individuals and cohorts:

$$\sum_s \int \ell_t^{j,s} dj = \sum_s n_t^s \ell_t^{j,s},$$

and compute  $A_t = \ell_t/\ell_t = \sum_s n_t^s z_t^s \ell_t^{j,s} / \sum_s n_t^s \ell_t^{j,s}$ , which becomes:

$$A_t = \frac{\sum_s n_t^s (z_t^{j,s})^{1+1/\varphi} (v_t^{j,s} \phi_t^{j,s} \lambda^{j,s})^{-1/\varphi}}{\sum_s n_t^s (z_t^{j,s})^{1/\varphi} (v_t^{j,s} \phi_t^{j,s} \lambda^{j,s})^{-1/\varphi}}.$$

The derivations show that the effect of heterogeneity by age can be summarized by four exogenous and foreseen paths of aggregate productivity  $\theta_t$ , to labor input  $A_t$ , to the consumption utility shifter  $\phi_t$ , and to the labor disutility shifter  $v_t$ .

## E Estimation with trend structural changes and the ZLB

This section details the estimation strategy tailored to my application with anticipated demographic shocks and where the zero lower bound is accounted for over the period 2009Q1 to 2015Q1.

## E.1 Solution method

A linear rational-expectations model can be written as:

$$\mathbf{A}x_t = \mathbf{C} + \mathbf{B}x_{t-1} + \mathbf{D}\mathbb{E}_t x_{t+1} + \mathbf{F}w_t, \quad (10)$$

where  $x_t$  is a  $n \times 1$  vector of state and jump variables and  $w_t$  is a  $l \times 1$  vector of exogenous variables. A solution to (10), following [Binder and Pesaran \(1995\)](#), is:

$$x_t = \mathbf{J} + \mathbf{Q}x_{t-1} + \mathbf{G}\varepsilon_t.$$

As in [Binder and Pesaran \(1995\)](#) and [Kulish and Pagan \(2012\)](#),  $\mathbf{Q}$  is solved by iterating on the quadratic expression:

$$\mathbf{Q} = [\mathbf{A} - \mathbf{D}\mathbf{Q}]^{-1} \mathbf{B}.$$

With  $\mathbf{Q}$  in hand, we compute  $\mathbf{J}$  and  $\mathbf{G}$  with:

$$\begin{aligned} \mathbf{J} &= [\mathbf{A} - \mathbf{D}\mathbf{Q}]^{-1} (\mathbf{C} + \mathbf{D}\mathbf{J}) \\ \mathbf{G} &= [\mathbf{A} - \mathbf{D}\mathbf{Q}]^{-1} \mathbf{F}. \end{aligned}$$

That is,  $\mathbf{J}$ ,  $\mathbf{Q}$  and  $\mathbf{G}$  are conformable matrices which are functions of the structural matrices  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ ,  $\mathbf{D}$  and  $\mathbf{E}$ .

In a model where agents have time-varying beliefs about the evolution of the model's structural parameters  $\mathbf{A}_t$ ,  $\mathbf{B}_t$ ,  $\mathbf{C}_t$ ,  $\mathbf{D}_t$  and  $\mathbf{F}_t$ , the solution becomes:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t w_t, \quad (11)$$

where  $\mathbf{J}_t$ ,  $\mathbf{Q}_t$  and  $\mathbf{G}_t$  are conformable matrices which are functions of the evolution of beliefs about the time-varying structural matrices  $\mathbf{A}_t$ ,  $\mathbf{B}_t$ ,  $\mathbf{C}_t$ ,  $\mathbf{D}_t$  and  $\mathbf{F}_t$  ([Kulish and Pagan, 2012](#)). They satisfy the recursion:

$$\begin{aligned} \mathbf{Q}_t &= [\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}]^{-1} \mathbf{B}_t \\ \mathbf{J}_t &= [\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}]^{-1} (\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1}) \\ \mathbf{G}_t &= [\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}]^{-1} \mathbf{E}_t, \end{aligned}$$

where the final structures  $\mathbf{Q}_T$  and  $\mathbf{J}_T$  are known and computed from the time invariant structure above under the terminal period's structural parameters.

Anticipated changes in the path of demographic shocks and the zero lower bound are anticipated changes to the model's structural parameters which can be handled by solution (11) (see [Jones, 2015a](#); [Kulish and Pagan, 2012](#), for details and a discussion).

## E.2 Kalman filter

Likelihood methods are used to estimate the parameters of the monetary policy rule and the parameters of the transitory shocks. For that, we need to filter the data, and owing to the linear structure of (11), we can use the Kalman filter, and exploit its computational advantages.

The model in its state space representation is:

$$x_t = \mathbf{J}_t + \mathbf{Q}_t x_{t-1} + \mathbf{G}_t \varepsilon_t \quad (12)$$

$$z_t = \mathbf{H}_t x_t. \quad (13)$$

The error is distributed  $\varepsilon_t \sim N(0, \Omega)$  where  $\Omega$  is the covariance matrix of  $\varepsilon_t$ . By assumption, there is no observation error of the data  $z_t$ . The Kalman filter recursion is given by the following equations, conceptualized as the predict and update steps. The state of the system is  $(\hat{x}_t, \mathbf{P}_{t-1})$ . In the predict step, the structural matrices  $\mathbf{J}_t$ ,  $\mathbf{Q}_t$  and  $\mathbf{G}_t$  are used to compute a forecast of the state  $\hat{x}_{t|t-1}$  and the forecast covariance matrix  $\mathbf{P}_{t|t-1}$  as:

$$\begin{aligned} \hat{x}_{t|t-1} &= \mathbf{J}_t + \mathbf{Q}_t \hat{x}_t \\ \mathbf{P}_{t|t-1} &= \mathbf{Q}_t \mathbf{P}_{t-1} \mathbf{Q}_t^\top + \mathbf{G}_t \Omega \mathbf{G}_t^\top. \end{aligned}$$

This formulation differs from the time-invariant Kalman filter because in the forecast stage the matrices  $\mathbf{J}_t$ ,  $\mathbf{Q}_t$  and  $\mathbf{G}_t$  can vary over time. We update these forecasts with imperfect observations of the state vector. This update step involves computing forecast errors  $\tilde{y}_t$  and its associated covariance matrix  $\mathbf{S}_t$  as:

$$\begin{aligned} \tilde{y}_t &= z_t - \mathbf{H}_t \hat{x}_{t|t-1} \\ \mathbf{S}_t &= \mathbf{H}_t \mathbf{P}_{t|t-1} \mathbf{H}_t^\top. \end{aligned}$$

The Kalman gain matrix is given by:

$$\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{H}_t^\top \mathbf{S}_t^{-1}.$$

With  $\tilde{y}_t$ ,  $\mathbf{S}_t$  and  $\mathbf{K}_t$  in hand, the optimal filtered update of the state  $x_t$  is

$$\hat{x}_t = \hat{x}_{t|t-1} + \mathbf{K}_t \tilde{y}_t,$$

and for its associated covariance matrix:

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t) \mathbf{P}_{t|t-1}.$$

The Kalman filter is initialized with  $x_0$  and  $\mathbf{P}_0$  determined from their unconditional moments, and is computed until the final time period  $T$  of data.

### E.2.1 Kalman smoother

With the estimates of the parameters and durations in hand at time period  $T$ , the Kalman smoother gives an estimate of  $x_{t|T}$ , or an estimate of the state vector at each point in time given all available information (see [Hamilton, 1994](#)). With  $\hat{x}_{t|t-1}$ ,  $\mathbf{P}_{t|t-1}$ ,  $\mathbf{K}_t$  and  $\mathbf{S}_t$  in hand from the Kalman filter, the vector  $x_{t|T}$  is computed by:

$$x_{t|T} = \hat{x}_{t|t-1} + \mathbf{P}_{t|t-1} r_{t|T},$$

where the vector  $r_{T+1|T} = 0$  and is updated with the recursion:

$$r_{t|T} = \mathbf{H}_t^\top \mathbf{S}_t^{-1} (z_t - \mathbf{H}_t \hat{x}_{t|t-1}) + (I - \mathbf{K}_t \mathbf{H}_t)^\top \mathbf{P}_{t|t-1}^\top r_{t+1|T}.$$

Finally, to get an estimate of the shocks to each state variable under this model's shock structure, denoted by  $e_t$ , we compute:

$$e_t = \mathbf{G}_t \varepsilon_t = \mathbf{G}_t r_{t|T}.$$

From these, we get an estimate of the structural shocks.

### E.3 Sampler

This section describes the sampler used to obtain the posterior distribution of interest. Denote by  $\vartheta$  the vector of parameters to be estimated and by  $\mathbf{T}$  the vector of ZLB durations that are observed each period. Denote by  $\mathcal{Z} = \{z_\tau\}_{\tau=1}^T$  the sequence of vectors of observable variables. The posterior  $\mathcal{P}(\vartheta \mid \mathbf{T}, \mathcal{Z})$  satisfies:

$$\mathcal{P}(\vartheta \mid \mathbf{T}, \mathcal{Z}) \propto L(\mathcal{Z}, \mathbf{T} \mid \vartheta) \times \mathcal{P}(\vartheta).$$

With Gaussian errors, the likelihood function  $L(\mathcal{Z}, \mathbf{T} \mid \vartheta)$  is computed using the appropriate sequence of structural matrices and the Kalman filter:

$$\log L(\mathcal{Z}, \mathbf{T} \mid \vartheta) = - \left( \frac{N_z T}{2} \right) \log 2\pi - \frac{1}{2} \sum_{t=1}^T \log \det \mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top - \frac{1}{2} \sum_{t=1}^T \tilde{y}_t^\top (\mathbf{H}_t \mathbf{S}_t \mathbf{H}_t^\top)^{-1} \tilde{y}_t.$$

The prior is simply computed using priors over  $\vartheta$  which are consistent with the literature.

The Markov Chain Monte Carlo posterior sampler has a single block, corresponding to the parameters  $\vartheta$ .<sup>42</sup> The sampler at step  $j$  is initialized with the last accepted draw of the structural parameters  $\vartheta_{j-1}$ .

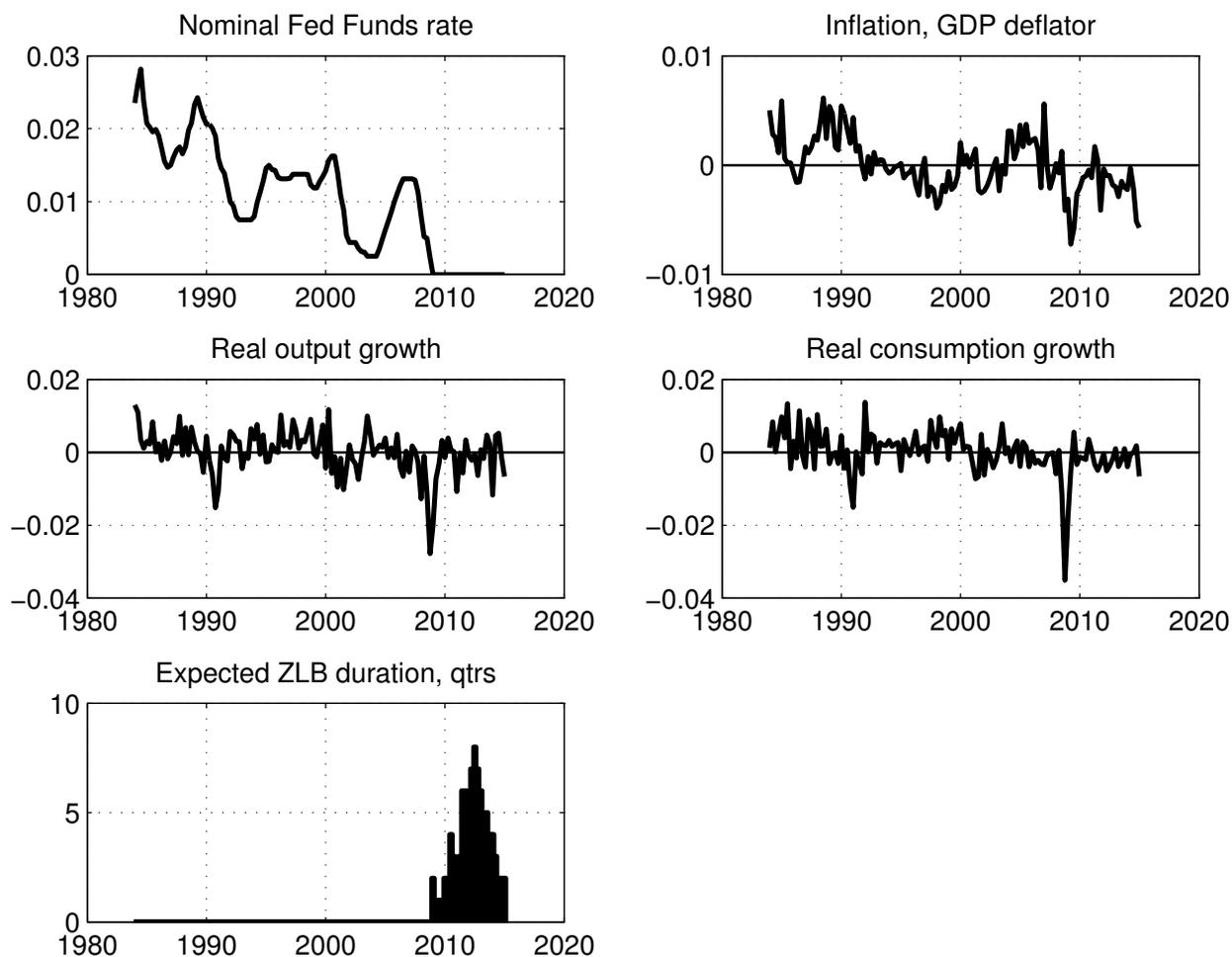
The block is a standard Metropolis-Hastings random walk. First start by selecting which structural parameters to propose a new value for. For those parameters, draw a new proposal  $\vartheta_j$  from a proposal density centered at  $\vartheta_{j-1}$  and with thick tails to ensure sufficient coverage of the parameter space and an acceptance rate of roughly 20% to 25%. The proposal  $\vartheta_j$  is accepted with probability  $\frac{\mathcal{P}(\vartheta_j \mid \mathbf{T}, \mathcal{Z})}{\mathcal{P}(\vartheta_{j-1} \mid \mathbf{T}, \mathcal{Z})}$ . If  $\vartheta_j$  is accepted, then set  $\vartheta_{j-1} = \vartheta_j$ .

## F Estimation results

### F.1 Data

In the baseline estimation, I use four observable series: real output growth per capita, real consumption growth per capita, GDP deflator inflation, and the Federal Funds rate. For the expected ZLB durations, I use Morgan Stanley's measure of the months until the first rate hike. These five series are plotted in [Figure F.1](#). Prior to estimation, I demean inflation, output and consumption growth.

<sup>42</sup>It is worth noting that one can estimate in addition to the structural parameters  $\vartheta$ , the expected zero lower bound durations can be estimated together with the structural parameters (as in [Kulish et al., 2014](#)),



**Figure F.1: Data used in estimation.** The nominal interest rate is not an observable variable between 2009 and 2015, while the ZLB duration becomes an observable variable during those quarters.

The expected ZLB durations follow a hump-shaped pattern, reaching a peak in 2012-13. This accords with the results of [Swanson and Williams \(2015\)](#).

## F.2 Calibrated parameters

To help with the identification of the shock parameters, I calibrate a small set of the structural parameters to values commonly used in the literature. The steady-state value of  $\theta$  is set at 11, which implies a steady-state markup over marginal costs of  $\theta/(\theta - 1) = 10\%$ . Applying the analysis of [Keen and Wang \(2005\)](#), who compare the Calvo price adjustment parameterization across different values of Rotemberg quadratic cost of price adjustment, I set  $\phi_p$  to 250, which in a Calvo pricing model would imply that with a steady-state markup of 10% over marginal costs, just less than 20% of firms each quarter reset their prices. With  $\beta$  and  $\varphi$  given by the lifecycle parameterization, and  $\theta$  and  $\phi_p$  calibrated, the choice of  $\phi_p$  implies that the coefficient on marginal costs in the Phillips curve is 0.1, which is consistent with low estimates

in which case an additional block is needed in the posterior sampler (see, for example, [Jones, 2015b](#)).

**Table 5:** Estimated parameters

| Parameter                  | Prior |      |        | Posterior |       |        |
|----------------------------|-------|------|--------|-----------|-------|--------|
|                            | Dist  | Mean | St Dev | Mode      | Mean  | St Dev |
| $\rho_r$                   | B     | 0.5  | 0.17   | 0.608     | 0.651 | 0.050  |
| $\phi_\pi$                 | N     | 2.0  | 0.15   | 2.382     | 2.384 | 0.203  |
| $\phi_y$                   | N     | 0.15 | 0.05   | 0.369     | 0.378 | 0.057  |
| $\rho_v$                   | B     | 0.5  | 0.17   | 0.046     | 0.112 | 0.049  |
| $\rho_\mu$                 | B     | 0.5  | 0.17   | 0.911     | 0.933 | 0.011  |
| $\rho_\theta$              | B     | 0.5  | 0.17   | 0.893     | 0.863 | 0.022  |
| $100 \times \sigma_v$      | IG    | 0.5  | 1      | 4.174     | 4.755 | 0.322  |
| $100 \times \sigma_\mu$    | IG    | 0.5  | 1      | 0.271     | 0.304 | 0.046  |
| $100 \times \sigma_\theta$ | IG    | 0.25 | 0.5    | 0.345     | 0.322 | 0.029  |
| $100 \times \sigma_r$      | IG    | 0.25 | 0.5    | 0.218     | 0.183 | 0.027  |

of the slope of the Phillips curve in the DSGE literature, and is the same value of the slope of the Phillips curve calibrated by [Ireland \(2004\)](#) in a similar model to the aggregate model here.

### F.2.1 Full estimation results

I estimate the model over the full sample 1984Q1 to 2015Q1. The prior distributions and posterior estimates for each estimation are given in [Table 5](#). The priors used for the structural parameters are standard in the literature. Prior beliefs suggest preference shocks are relatively large relative to other shocks (including markup shocks, which I scale by  $1/\phi_p$ , the coefficient on markup shocks in the linearized model). Permanent and transitory technology shocks are given the same prior variance. The policy rule parameters are given standard prior distributions (see, for example, [Smets and Wouters, 2007](#)).

[Figure F.2](#) plots  $R^2$  Gelman chain diagnostics for the baseline estimates from 1984Q1 to 2015Q1, and illustrate that the estimated posterior distributions lie comfortably below 1.1, commonly used as a value indicating convergence of the posterior distributions (see, for example, [Bianchi, 2013](#)).

The prior and posterior distributions for the estimated parameters are plotted in [Figure F.3](#).

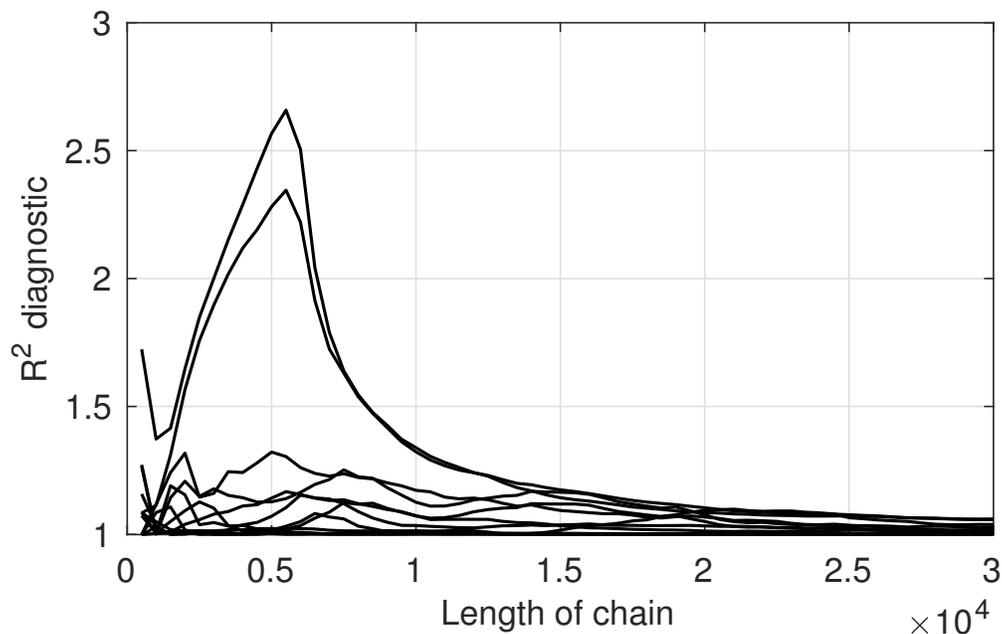
## G Data sources

### G.1 Data sources for OLG model

This section details the data series used for calibration of the lifecycle model.

**Current Population Survey** I use the Current Population Survey to get estimates of labor force participation rates by age in the calibration of the disutility of providing labor.

**Census/American Community Survey** I use Census and American Community Survey extracts from IPUMS-USA to compute the experience-productivity profiles following [Elsby](#)



**Figure F.2: Gelman chain diagnostic.** The black lines are the calculated  $R^2$  statistic for each structural parameter.

and Shapiro (2012).

**Social Security Administration** I use Social Security Administration estimates and forecasts for mortality rates  $\gamma_t^s$ . These numbers are directly fed into the model as anticipated paths.

**BLS-Multifactor Productivity Program** I use the BLS-MFP data to construct a measure of the real interest rate from observed capital-output ratios.

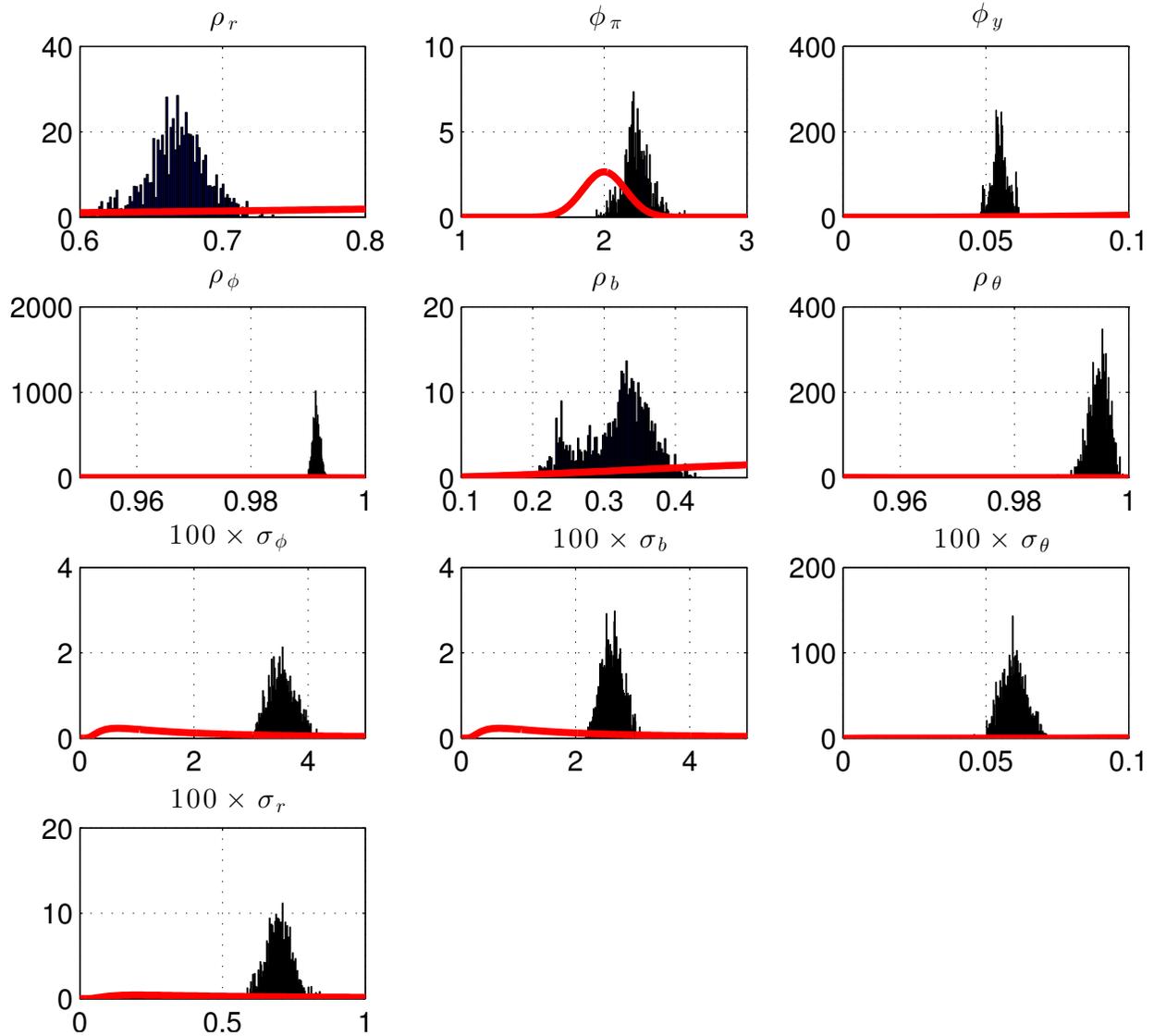
From the BLS website: “Capital input data—service-flows of equipment, structures, intellectual property products, inventories, and land. BLS measures of capital service inputs are prepared using NIPA data on real gross investment in depreciable assets and inventories. Labor input data—hours worked by all persons engaged in a sector—is based on information on employment and average weekly hours collected in the monthly BLS survey of establishments and the hours at work survey. Labor composition data are based on March supplements to the Current Population Survey.”

## G.2 Data sources for estimating the New Keynesian model

I use data on output, consumption, inflation, and interest rates.<sup>43</sup> Construction of the data series follows Smets and Wouters (2007). The codes for each raw data series are as follows:

- Real Gross Domestic Product, 3 Decimal (GDPC96). Current, \$.
- Gross Domestic Product: Implicit Price Deflator (GDPDEF). Index, 2009=100.

<sup>43</sup>A public version of the data list corresponding to the Smets and Wouters (2007) series can be obtained at <https://research.stlouisfed.org/pdl/803>.



**Figure F.3: Estimated posterior distributions.** This figure shows the priors and posteriors of the estimated parameters.

- Personal Consumption Expenditures (PCEC). Current, \$.
- Total population (CNP16OV), Thousands of Persons.

To map these data series to the model variables, I do the following transformations.

1. Construct the series LNSindex, which is an index of CNP16OV where 1992Q3=1. I adjust the CNP16OV series to account for breaks in the series each January, due to revisions from updated Census reports, which can be substantial. To do this, I impute an estimate of each January's monthly change in population and construct an estimate of the revised change in population from the actual change to the constructed imputed change. I then distribute that revised change in population across the preceding 12 months.

2. Construct the series CE16OVIndex, which is an index of CE16OV where 1992Q3=1.
3. Output =  $Y_t = \ln(\text{GDPC96} / \text{LNSindex}) * 100$ . Then compute the percentage change in output as an observable,  $\ln Y_t - \ln Y_{t-1}$ .
4. Inflation =  $\Pi_t = \ln(\text{GDPDEF} / \text{GDPDEF}(-1)) * 100$ .
5. Consumption =  $C_t = \ln((\text{PCEC} / \text{GDPDEF}) / \text{LNSindex}) * 100$ . Then compute the percentage change in consumption as an observable,  $\ln C_t - \ln C_{t-1}$ .

The interest rate is the Federal Funds Rate.

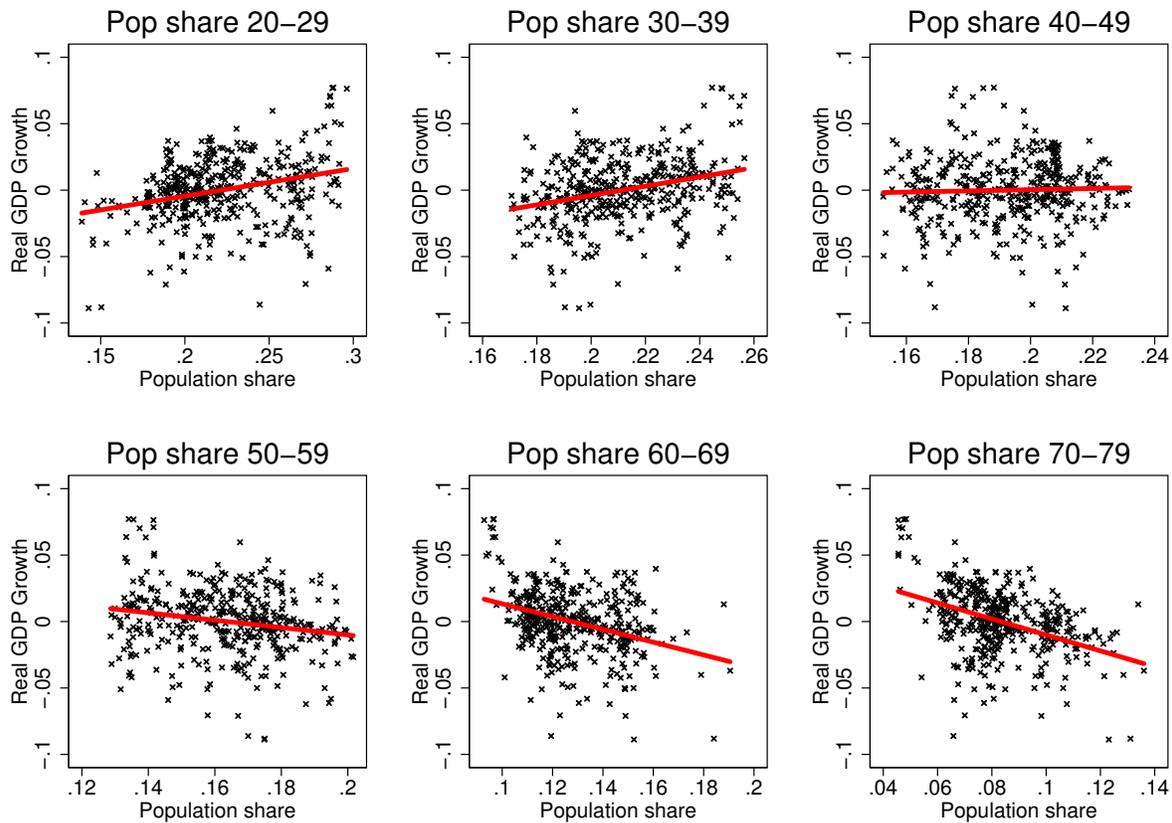
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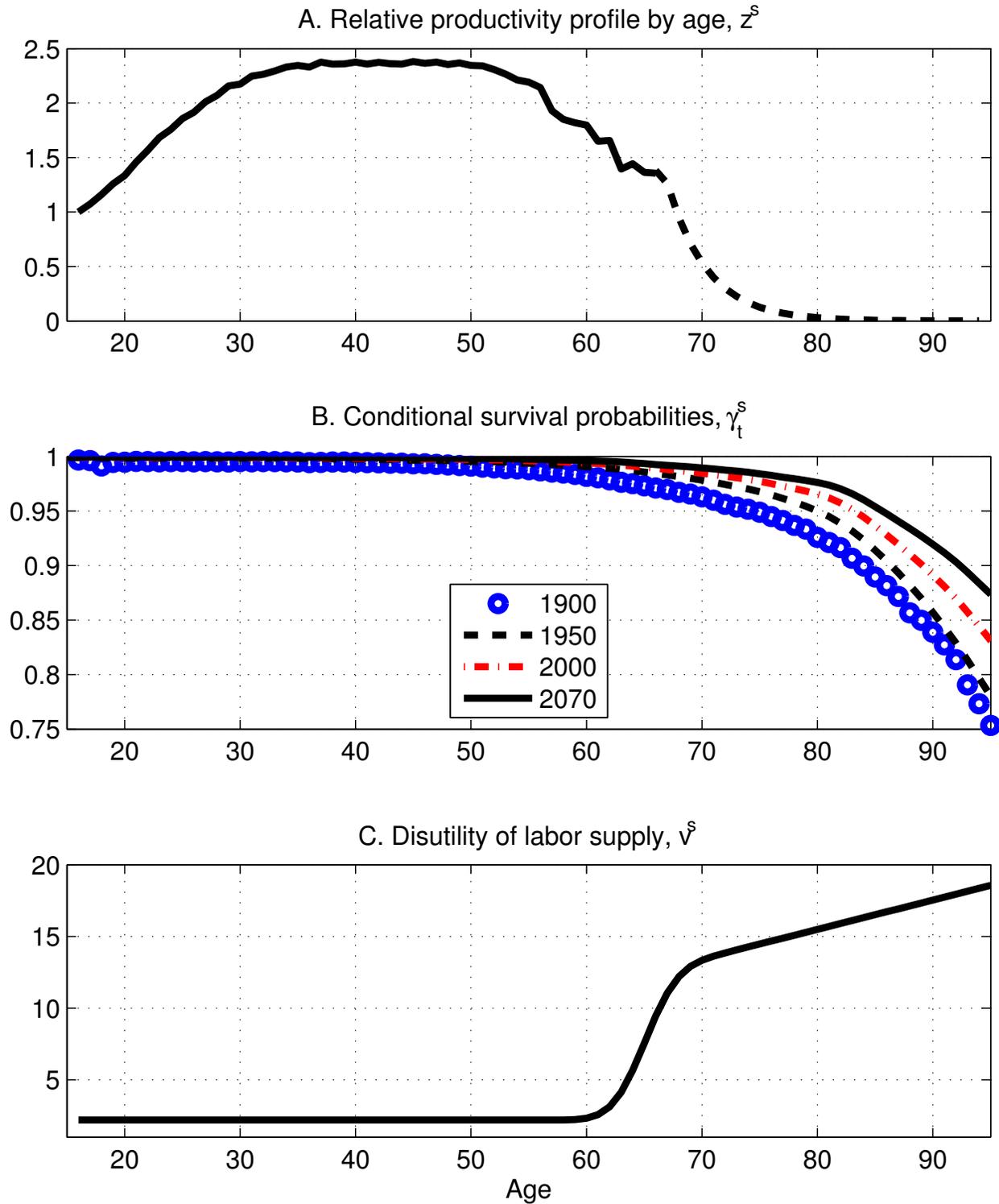
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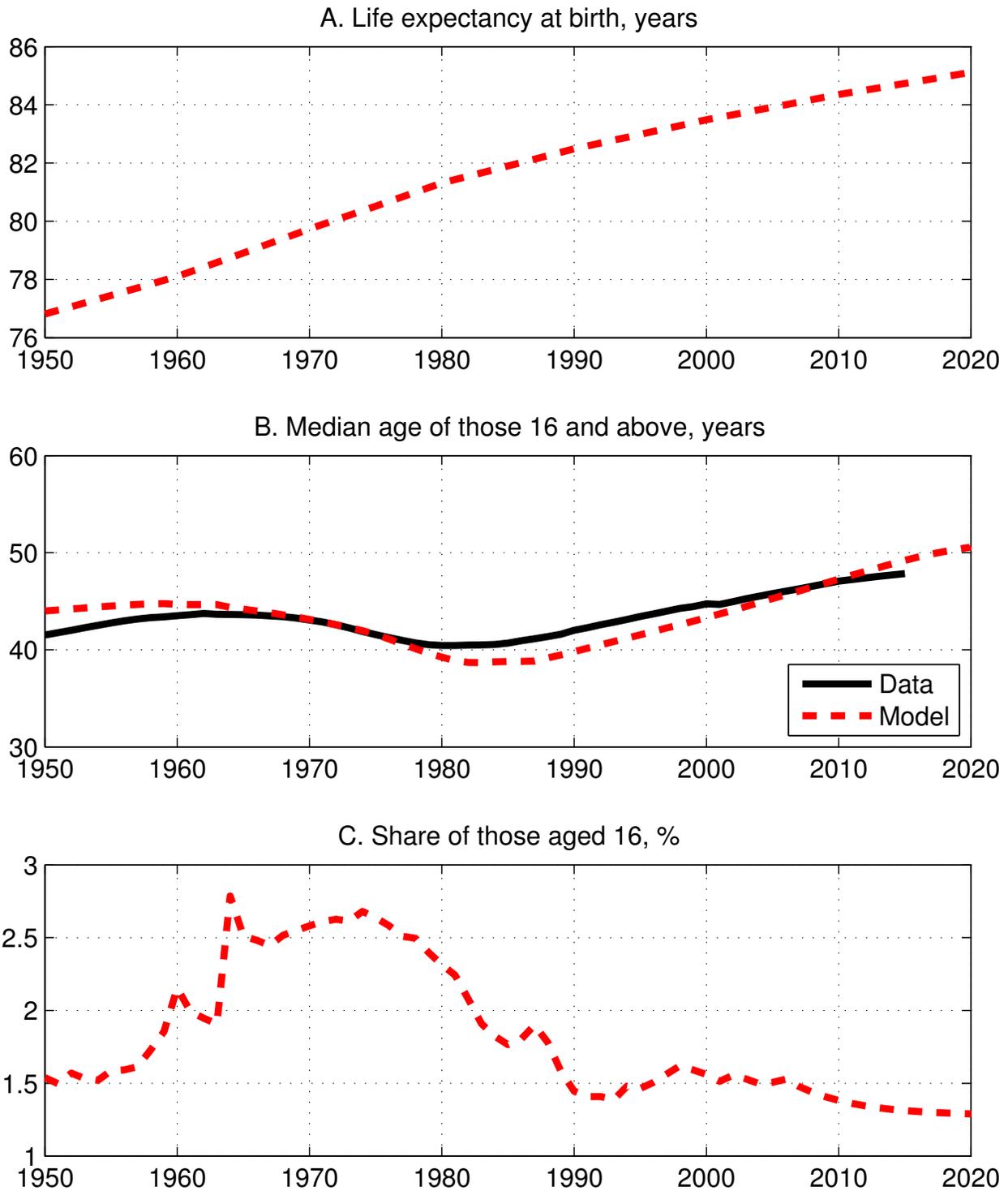
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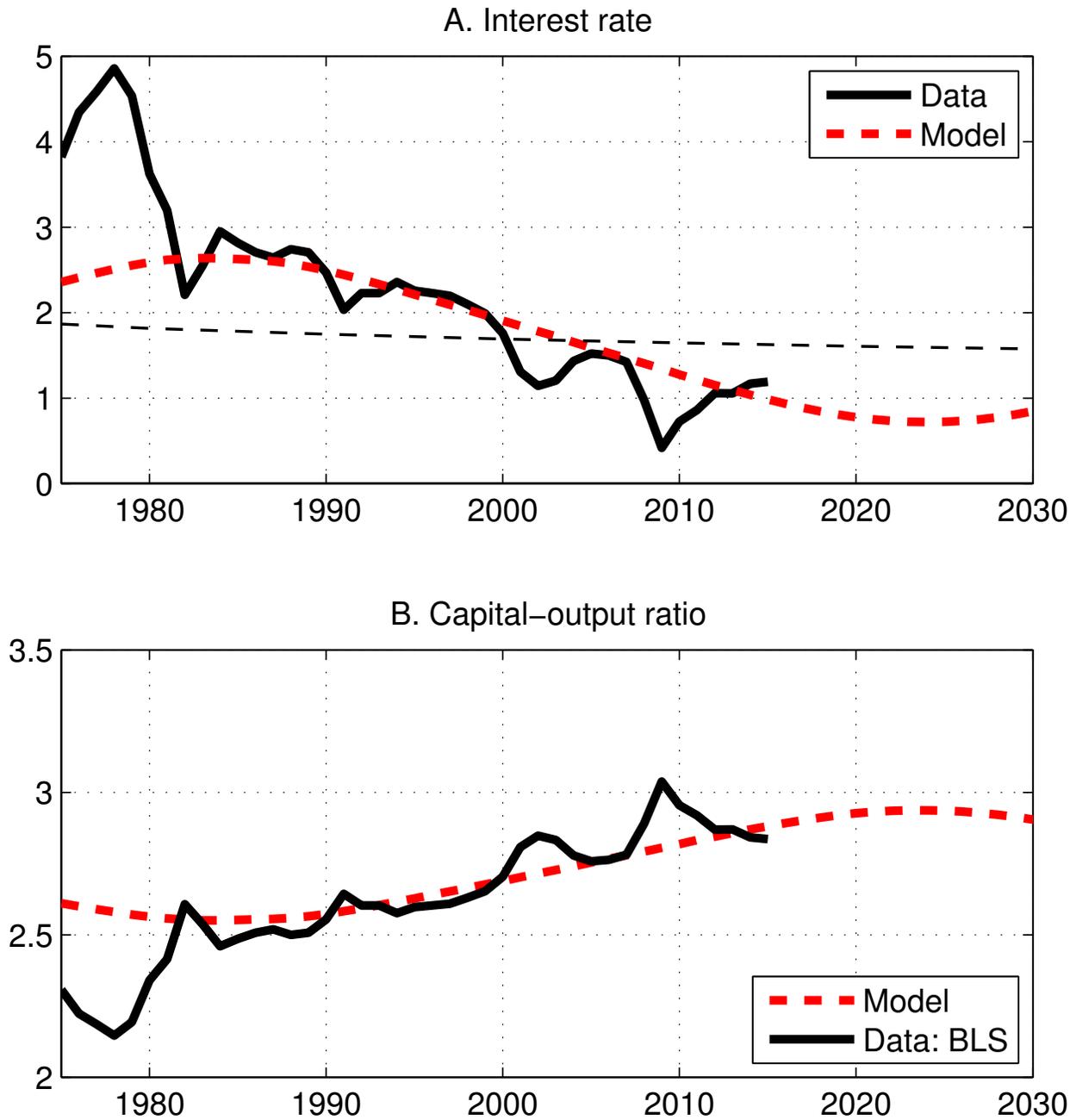
**Figure 1: Real GDP growth and population share.** This figure shows cross country evidence for a relationship between real GDP growth and the age-composition of the population. The countries included are: Australia, Canada, France, Italy, Japan, New Zealand, the United Kingdom, and the United States, over the years 1950 to 2015. The data are from the United Nations Population Division and the Federal Reserve Economic Database.



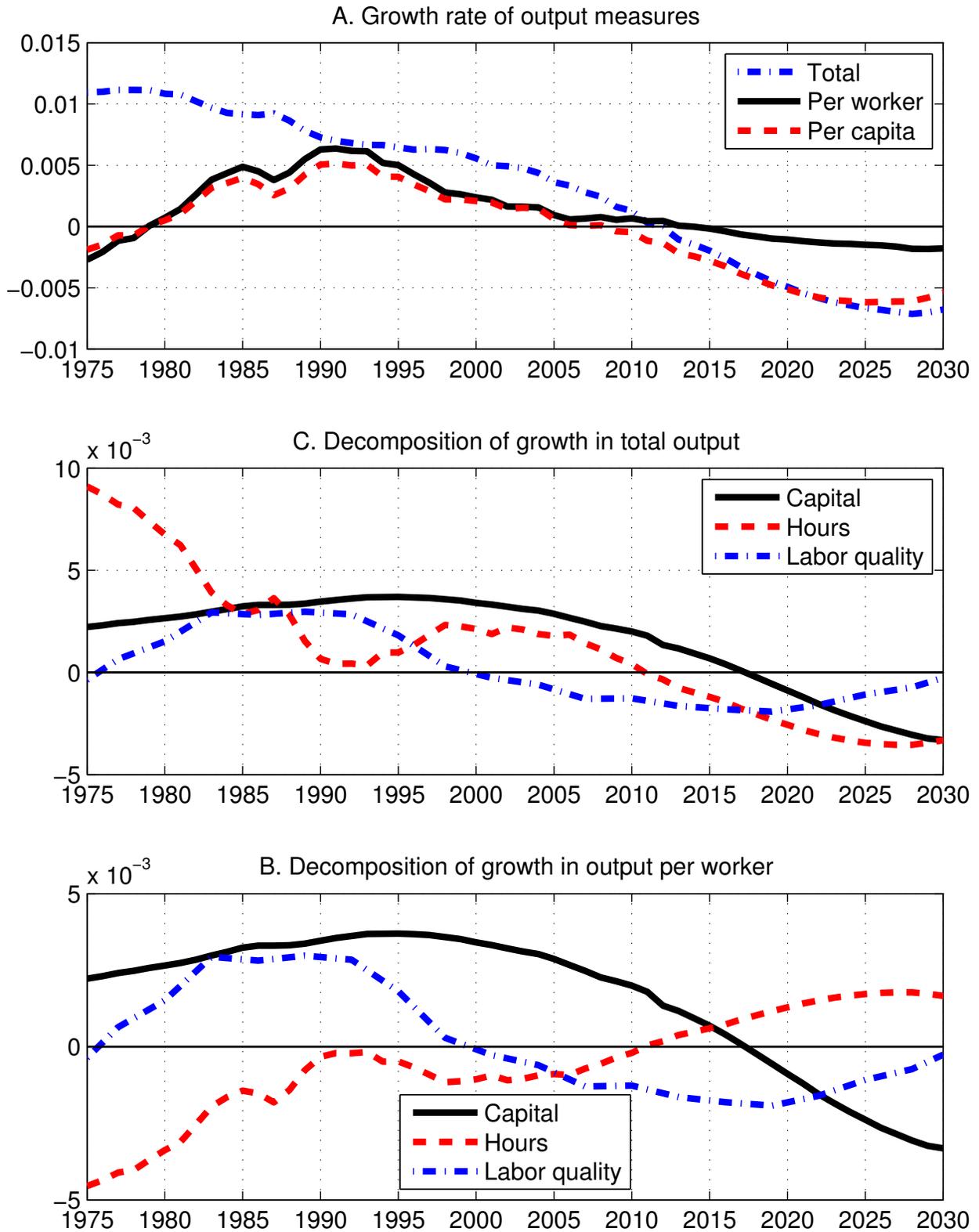
**Figure 2: Calibrated lifecycle profiles.** This figure shows the calibrated lifecycle profiles for the age-productivity parameters, the conditional survival probabilities and the disutility of labor supply.



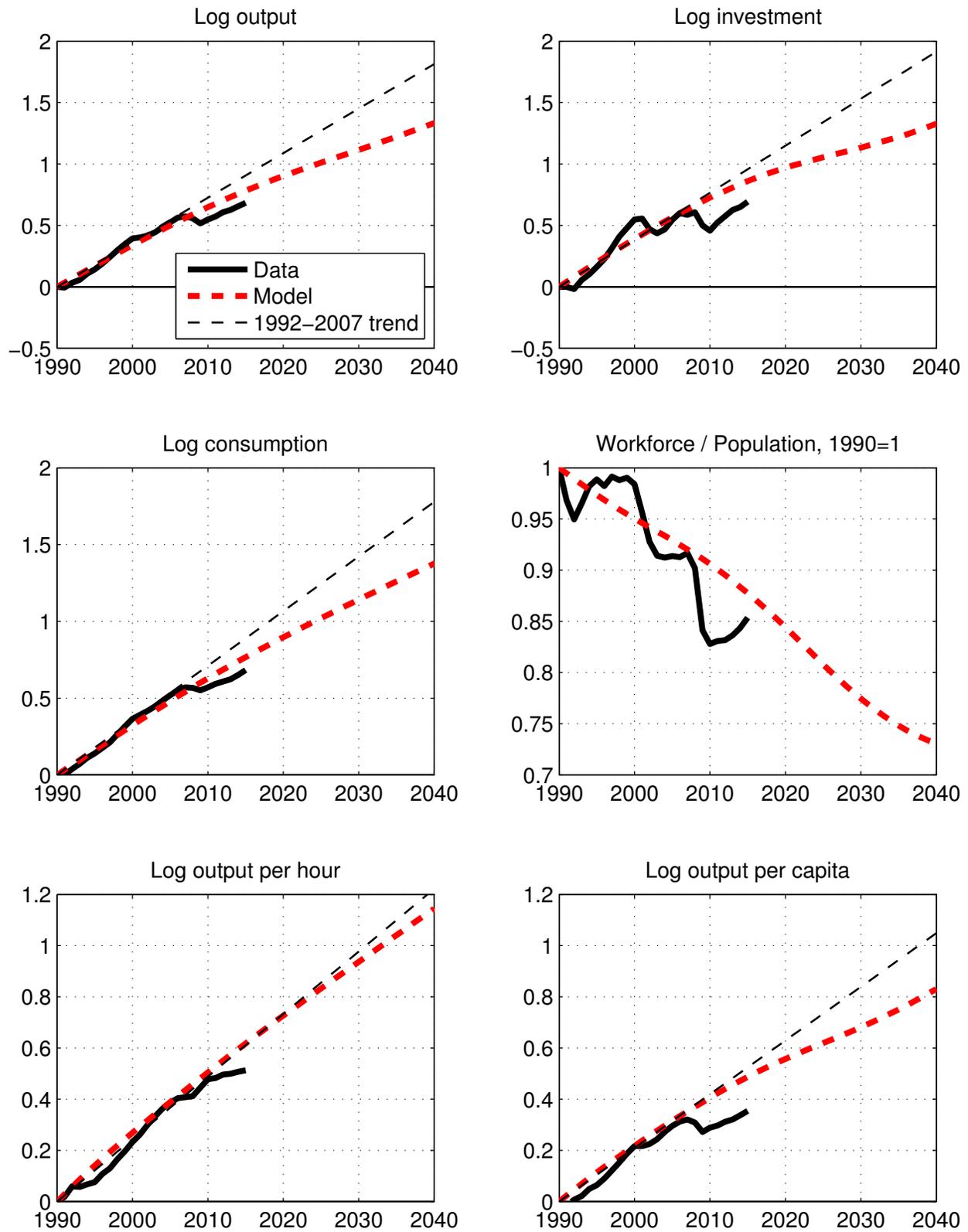
**Figure 3: Demographic trends in model.** This figure shows three summary measures the exogenous demographic variables. Life expectancy conditional on being 16 rises by about 10 years while the median age of those 16 and above decreases from 1960 to 1980 and rapidly increases thereafter. The share of those age 16 is used to calibrate the initial population each period.



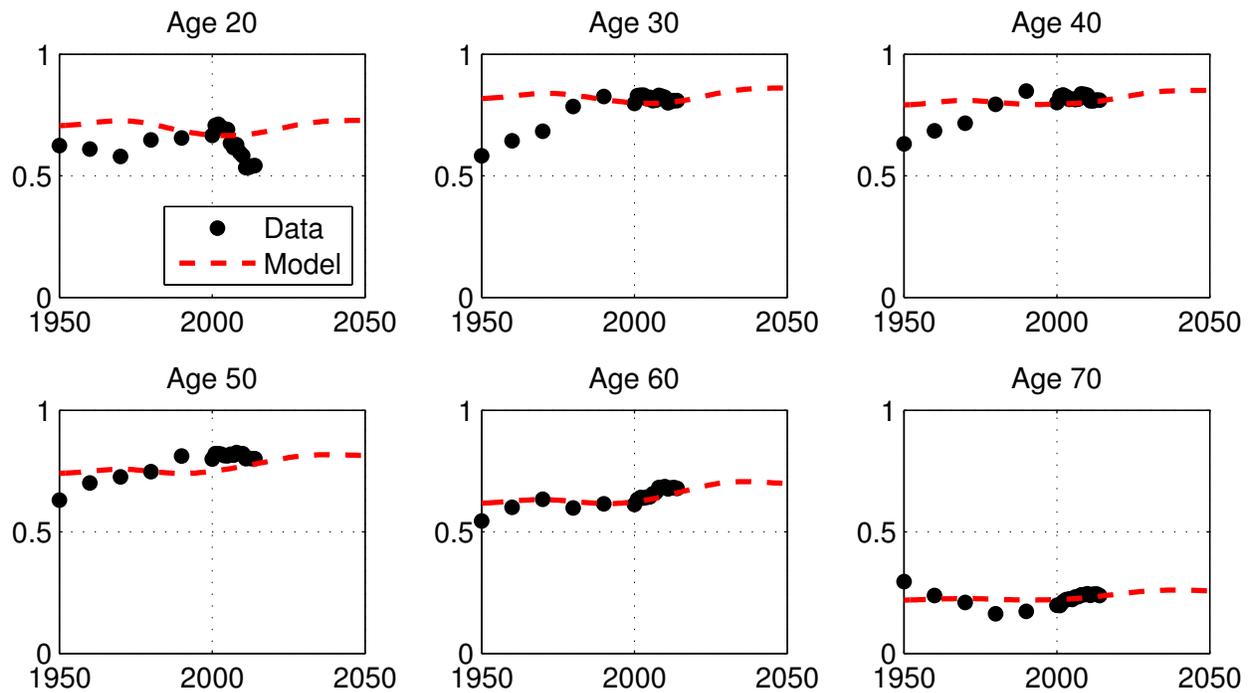
**Figure 4: Capital-output ratio and real interest rate.** Panel A shows changes in the output-capital ratio in the model against the ratio of measure from the Multifactor Productivity Program. Panel B plots the model's interest rate in the full demographic transition. Peak-to-trough between 1985 and 2010, the interest rate falls about 1.5% points.



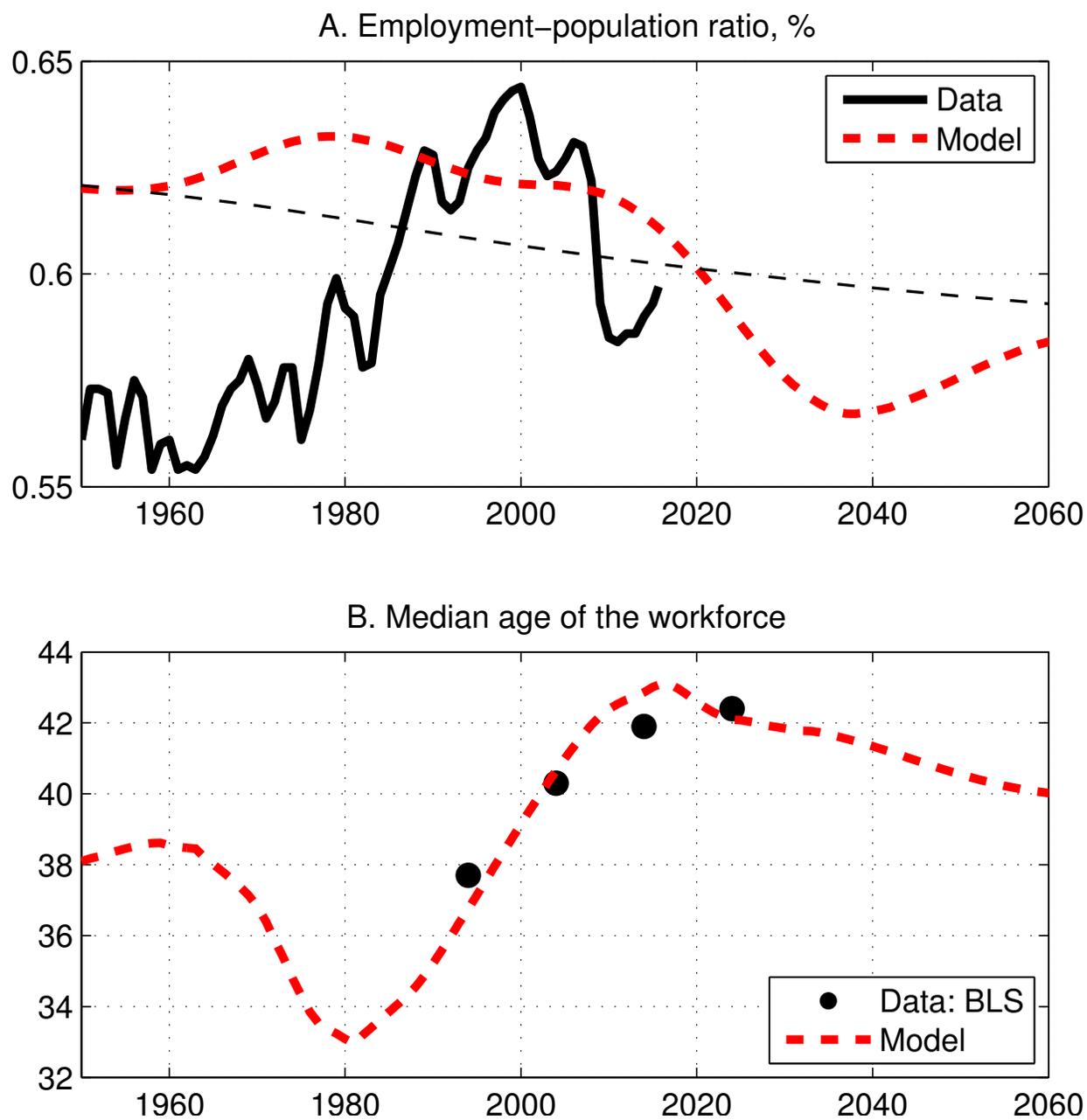
**Figure 5: Measures of output growth and their decomposition.** This figure plots different measures of output growth and the decomposition of output per worker and total output into changes in factors of production.



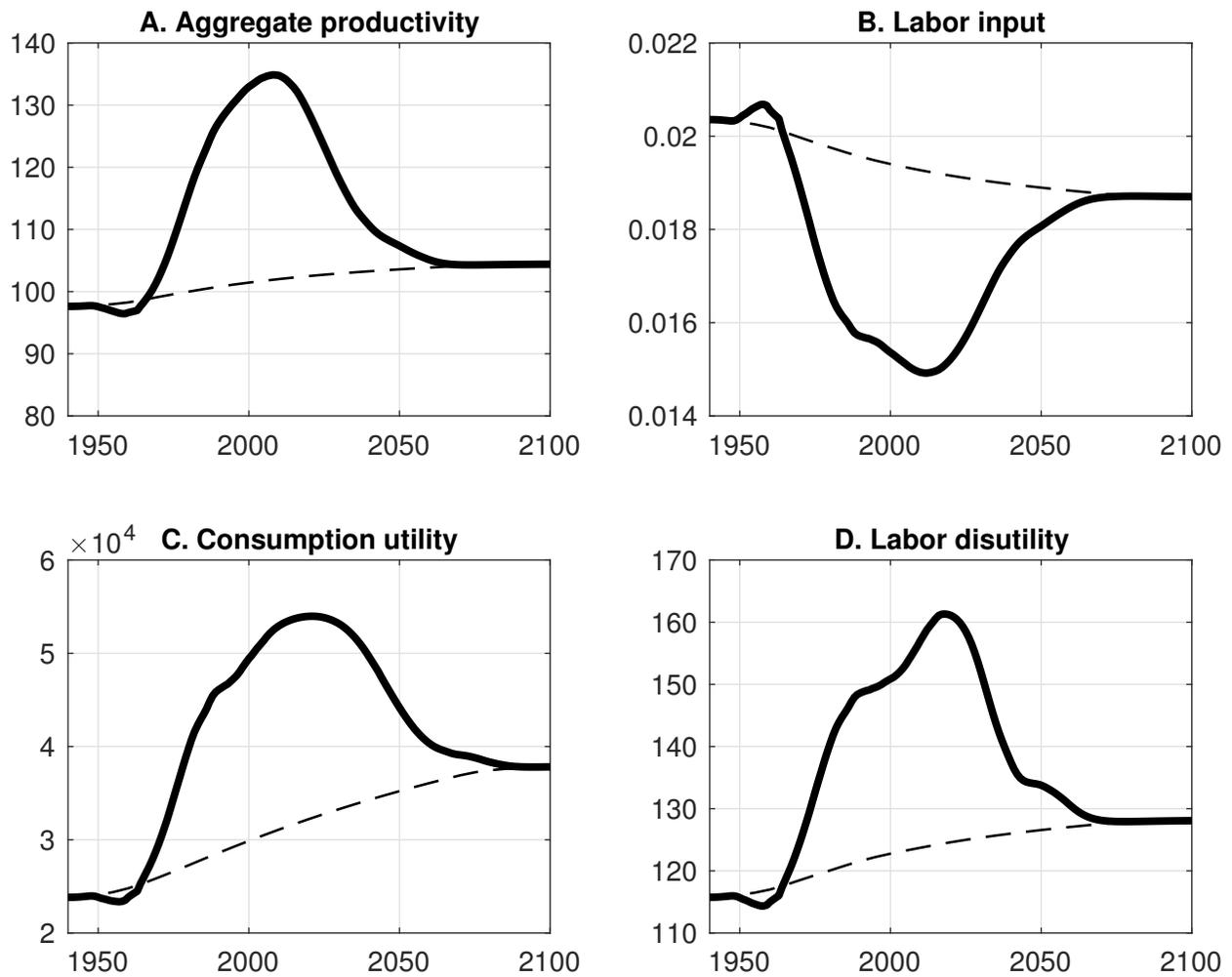
**Figure 6: Trends of key variables.** This figure plots the model's trend paths for some key variables against the observed indexes and a pre-crisis trend path, compared on the average growth rate of each variable from 1992 to 2007.



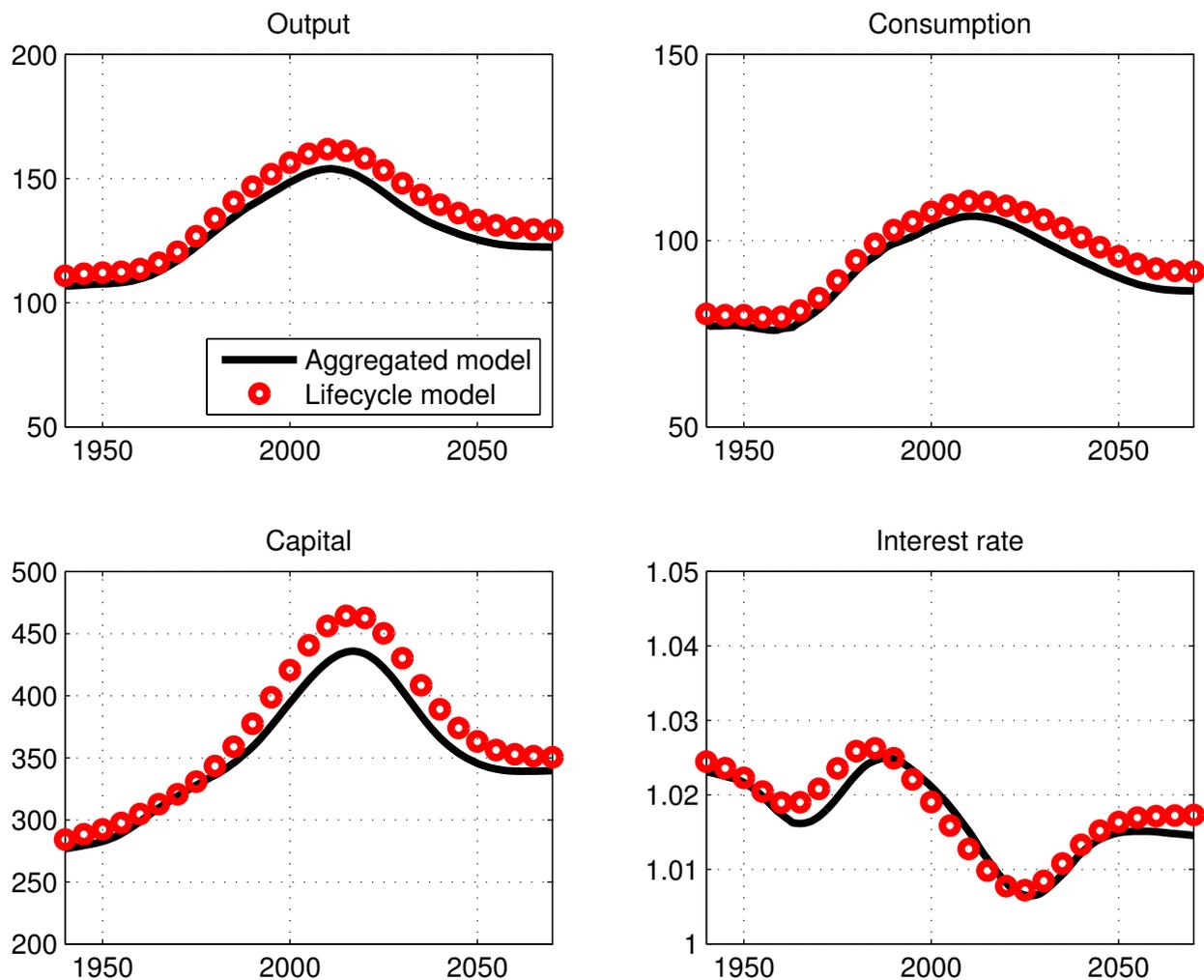
**Figure 7: Labor force participation by age.** This figure shows the fraction of the labor endowment chosen by workers at each age in the model against the labor force participation rates observed in censuses and American Community Surveys.



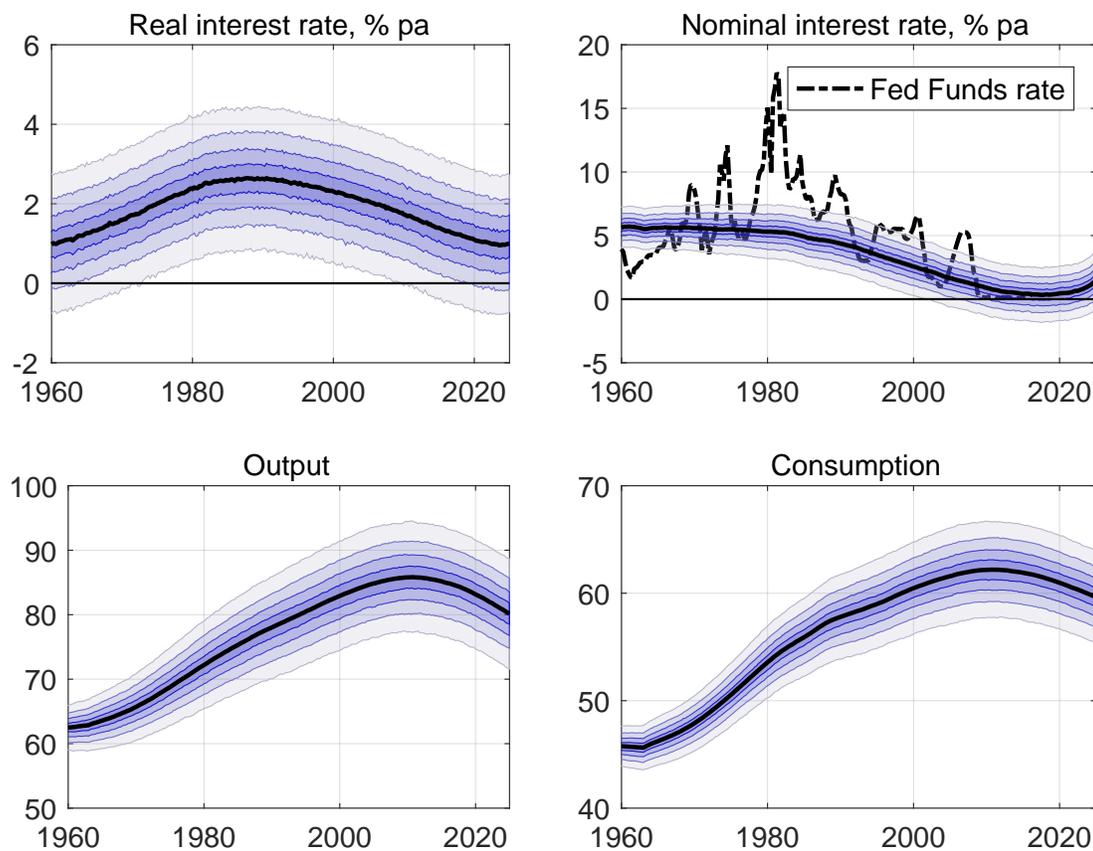
**Figure 8: Employment–population ratio.** This figure shows total employment–population ratios in the model and as observed (Panel A), and the median age of the workforce (Panel B).



**Figure 9: Shocks.** This figure shows the paths of the aggregate anticipated shocks that summarize the aggregate effects of heterogeneity in the lifecycle model. Aggregate productivity (aggregate supply, panel A) increases with the population size, peaking around 2010. The consumption utility (aggregate demand, panel C) also increases with the population size, peaking around 2025.



**Figure 10: Comparison of aggregated and lifecycle model.** This figure compares the paths for aggregate variables from the full lifecycle model solution and the approximation under the linearized framework with aggregate shocks and the fully anticipated paths of the average mortality rate and labor income tax rate.



**Figure 11: Stochastic simulations of the business cycle model during demographic transition.** This figure shows fancharts of simulated paths around the demographic