Physics 213a-2011 Coverage for Final Exam

This exam will be truly cumulative, but there will be extra emphasis on the material since exam 2, since you've not yet been tested on it:

Reading: Chapter 6: 6.5 (earlier sections covered on exam 2), Chapter 7: All
Chapter 8: All, Chapter 9: 9.1-9.4, 9.5 from start through (9.5.14), 9.7, 9.10, 9.12
Chapter 10: pp. 343-4, first two paragraphs of 10.2, 10.5, 10.8, start of 10.9 through p. 373

Class sessions: 10-22-10 through end of semester

Assignments: 7-10

Topics (topics in boldface are somewhat more important, but don't ignore the others):

All topics from Exams 1 and 2

Matrix operations in Mathematica:

- **Eigenvalues, Eigenvectors, entering square matrices, entering row/column matrices, matrix multiplication, normalizing eigenvectors**

Beaded string

- How to find the total force on a particular mass on the string
- boundary conditions
- **For finite or infinite *N*, normal modes are standing waves (be sure to include the equations for the frequencies of the normal modes and the eigenvectors.)**
- **There really are only *N* normal modes (e.g. $y_{N+2} = y_N$) and why this is so mathematically and physically**
- **Normal modes are mutually orthogonal (& what "orthogonality" means in this context)**
- Number of nodes = n-1
- The normal modes of this system are also called "standing waves"
- **The boundary conditions quantize the frequencies**
- Longitudinal oscillations
- What happens when the ends of the chain of oscillators are unconstrained (mostly for longitudinal oscillations)

Continuous string

- How this is the same as the beaded string in the limit $N \to \infty$
- **equations for frequencies of normal modes and eigenvectors (i.e. initial shapes)**

Normal mode analysis for beaded string, continuous string

- **Any behavior of a system can be expressed as a superposition of normal modes, each with its own amplitude and phase**
- How this "normal mode expansion" way of thinking contains the same information as the more conventional way of thinking (in which we consider the position and velocity of each object in the system)
- Fact that each of the "basis vectors" in the expansion is orthogonal to each of the others.
- Meaning of "orthogonal", "orthonormal", "basis function", and "complete"
- **Definition of the inner product for continuous systems**
- **How to find the complex coefficients in the normal mode expansion, given the initial positions and velocities.**
- **How to then predict the behavior of the system for later times**

Generic orthonormal function analysis

- **Given a complete set of basis functions, and a target function, be able to calculate the expansion coefficients needed to synthesize the target function from the basis functions**

Fourier analysis

- **Difference in the definitions of *k* for Fourier analysis and for Normal mode analysis**
- **Equation for the Fourier expansion of *y(x)* in terms of sines, cosines, and constant**
- Fact that each of these "basis vectors" is orthogonal to each of the others.
- **How to compute the values of the a_n's and b_n's for a given *y(x)***
- Version in terms of $\cos(k_n x + \varphi_n)$

Complex exponential version
Fact that the Fourier expansion contains the same info. as *y(x)*, but viewed in a different (and often very useful) way
Fact that you can do a Fourier expansion of *y(t)*, and what the formulas would be for this case.
Hilbert Space picture: any periodic function is represented by a point in this space, and the sinusoidal functions with the same periodicity are the "unit vectors".
k-space picture: the function can be fully specified by showing it in real space (*i.e.* showing $y(x)$) or by showing it in *k*-space (*i.e.* showing the a_n's and b_n's as a function of *k*).

Fourier transforms
Represent the limit of Fourier series for $\lambda \to \infty$
Know the formulas for Fourier transforms, be ready to use them, and to think about them in terms of the Hilbert space picture.
Be prepared to discuss the connection between the width of a pulse in real space and the width of the same pulse in *k*-space

Discrete Fourier Transform (DFT)
Aliasing, maximum wiggliness, and the Nyquist limit
The frequency of any sinusoid can be shifted up or down by f_{sample} without affecting the DFT or the inverse DFT
Windowing – why it's needed
Dependence of frequency resolution on length of sampling window
Dependence of maximum meaningful frequency on interval between samples
Order of frequencies in the usual presentation of the DFT (e.g. fig. 8.6.3)

Wave propagation (general)
The wave equation
Solutions to the wave equation are of the forms *y(x-vt)* and *y(x+vt)*
These forms correspond to (respectively) right-moving and left-moving waves with speed *v*
phase velocity $v_p \equiv \omega/k$ is independent of ω for ropes, em waves in vacuum, sound, and any other wave in a non-dispersive medium (*i.e.* one with a linear dispersion relation)
equations for *v* for rope waves, em waves, transmission line waves
Because the wave equation is linear, waves can superpose. In particular, pulses can "pass through" each other without permanent effects.
Superposing equal amplitude, equal wavelength sinusoidal waves traveling in opposite directions produces a standing wave.
Power: proportional to amplitude^2

Rope waves
Connection between a motion imposed on the end of a rope and the resulting wave (e.g. problem 9.2)

Group velocity
what "dispersion relation" means, and why it has that name
Meaning of "carrier wave", "sideband"
Multiplying a complicated low-frequency signal by a carrier wave shifts the Fourier components of the signal from being centered on zero frequency to being centered on the carrier frequency
definition of group velocity
physical interpretation of group velocity
how to tell whether the group velocity or the phase velocity is higher, given the dispersion relation (either graphically or in an equation)
requirement on the dispersion relation for the envelope to propagate without changing its shape

Electromagnetic waves in vacuum
Poynting vector, understanding of the meaning of intensity (i.e. power/area)
rms amplitudes
Qualitative understanding of how an em wave "supports itself" even in a vacuum

For em plane waves, $E = cB$

How to apply Maxwell's equations to test whether a particular combination of electric and magnetic fields is physically allowed. (I would only expect you to be able to do this for simple field configurations, such as the plane waves we've treated in class and on the homework. For example, I might ask you to check whether a field of the form $\vec{E} = x^2\hat{x} \quad \vec{B} = \sin(x)\hat{y}$ is allowed by Faraday's law.)

Meaning of "plane wave"

Electrical transmission lines (e.g. coax cables)

Discrete component model for transmission line (ladder of inductors and capacitors)

Isomorphism with em waves in vacuum

$v = \dfrac{1}{\sqrt{C_o L_o}}$

$Z \equiv \dfrac{I}{V}$ for right-moving waves; positive I is defined as going to the right

$Z = \sqrt{L_o / C_o}$ = 50 Ω for standard coax

***Z* is real ⇒ *I* and *V* waves are in phase for right-moving waves (but out of phase for left-moving waves)**

Meaning of *Z*

Why an ohmmeter connected between the inner and outer conductors of a 1 m long coax reads $R = \infty$ instead of R = 50 Ω; why an ohmmeter connected to a very long coax would read 50 Ω very briefly.

How to eliminate reflections

Reflection and transmission at boundaries for rope waves (qualitative only) & transmission line waves

Use of boundary conditions to find reflection and transmission amplitudes in terms of incident amplitude. **Be ready to reproduce the argument for transmission line waves, starting from boundary conditions and resulting in the expressions for the reflected and transmitted amplitudes for the voltage and current waves.**

Equations for reflection and transmission amplitudes for voltage and current

Shape of reflected pulse: Always flipped left-right (⇔ causality)
Inverted or non-inverted depending on Z_1 & Z_2
Amplitude changed, depending on Z_1 & Z_2

Shape of transmitted pulse: x-scale changed by v_1 / v_2
y-scale: non-inverted, but height can be larger or smaller than incident, depending on Z_1 & Z_2
Sinusoids: λ, v change but ω remains the same

For rope waves: type of reflection for termination in a wall or massless ring

Optics (If we get to it. Easy problems only)

Snell's law

Total internal reflection

Miscellaneous

Kronecker delta function

Dirac delta function

General: The best way to study is usually to work additional problems. There are many from the book that I've not assigned that would be good candidates for this; don't just do the easy ones. You should also go over your lecture notes and assignments.

As you may have noticed from previous physics exams in this course and others, your professors are interested in how well you can apply what you've learned to a new situation. Therefore, for example, you might to get questions about reflections and transmissions at boundaries for systems other than ropes and transmission lines. Don't panic on such questions; read the question carefully twice, then apply what you've learned. Check your results qualitatively by analogy with more familiar systems. If you can't see how to proceed at all, work the analogous problem for a more familiar system, then see whether you can translate the results to the less familiar system which is the subject of the problem.

Mathematica commands and functions you should be familiar with

In addition to the basics (e.g. integration, definite integration, summation, differentiation, defining functions, defining operators, Sin, Cos, etc.), you should be familiar with the following

Functions relating to complex numbers:
Abs
Arg
Re
Im

Functions relating to lists:
ListPlot
Table

Functions relating to eigenvalue equations:
Eigenvalues
Eigenvectors

Functions relating to matrices and vectors:
Inputing square matrices and row/column vectors, e.g. **y0 = {0.2,0.3, −0.1}** (same format for both row and column vectors)
Taking products of matrices, taking inner product of two vectors, e.g. y0.y1

Functions relating to plotting functions (as opposed to lists):
Plot
LogPlot

Misc (but important!):
Animate
GridLines
PlotRange
Piecewise
Solve
Simplify
FullSimplify
$Assumptions
Clear
Esc-el-Esc (gives the "element of" symbol: ∈)