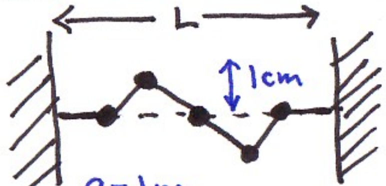
Physics 213-2011 Class 25 Wednesday 11-1-11 Summary

An example of normal mode analysis for the beaded string



$a = 1\,m$
$m = 1\,kg$
$T = 1\,N/m$

All initial velocities $= 0$

Given this initial condition, what is

$$|y(t)\rangle = \begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \end{pmatrix} ?$$

A) Express $|y(t)\rangle$ as a superposition of normal modes:

$$|y(t)\rangle = \mathrm{Re}\left[\sum_{n=1}^{N} C_n e^{i\omega_n t} |e_n\rangle\right]$$

We have seen that the entries Y_j in the eigenvector

$$|e_n\rangle = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \end{pmatrix}$$ are proportional to $\sin k_n x_j$,

where $k_n = \frac{n\pi}{L}$, x_j = horizontal position of bead j $= ja$

Problem 7.6 $\Rightarrow$ the correct normalizing factor is $\sqrt{\frac{2}{N+1}}$ $\Rightarrow$

$$|e_n\rangle = \sqrt{\frac{2}{N+1}} \begin{pmatrix} \sin k_n x_1 \\ \sin k_n x_2 \\ \vdots \end{pmatrix}$$

$$k_n x_j = \frac{n\pi}{(N+1)a} ja = \frac{n\pi j}{(N+1)}$$

Our example: $N = 5$

$$\Rightarrow |e_1\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} \sin\frac{\pi}{6} \\ \sin\frac{2\pi}{6} \\ \sin\frac{3\pi}{6} \\ \vdots \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \\ 1 \\ \sqrt{3}/2 \\ 1/2 \end{pmatrix}$$

So, we know what the eigenvectors & eigenvalues are. We could also find them by our old methods:

$$\rightarrow \hat{A} = \frac{T}{am} \begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

(rest of summary is in Mathematica notebook)