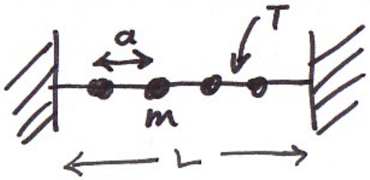
The beaded string



$\dashrightarrow$ $\omega_n = \sqrt{2}\,\omega_A \sin \frac{k_n a}{2}$

where the wavenumber $k_n \equiv 2\pi/\lambda_n$

This is a nonlinear dispersion relation

** $\boxed{\text{Dispersion relation: the relation between } k = 2\pi/\lambda \text{ and } \omega = 2\pi/T}$ **

Note: the boundary conditions (that $y \to 0$ at $x = 0, L$) required $\lambda_n = 2L/n$. This in turn places restrictions on k_n and, through the dispersion relation, on ω_n

$\dashrightarrow$ *** $\boxed{\text{The boundary conditions quantize } \omega}$ ***

This explains quantization of electron energies in atoms.

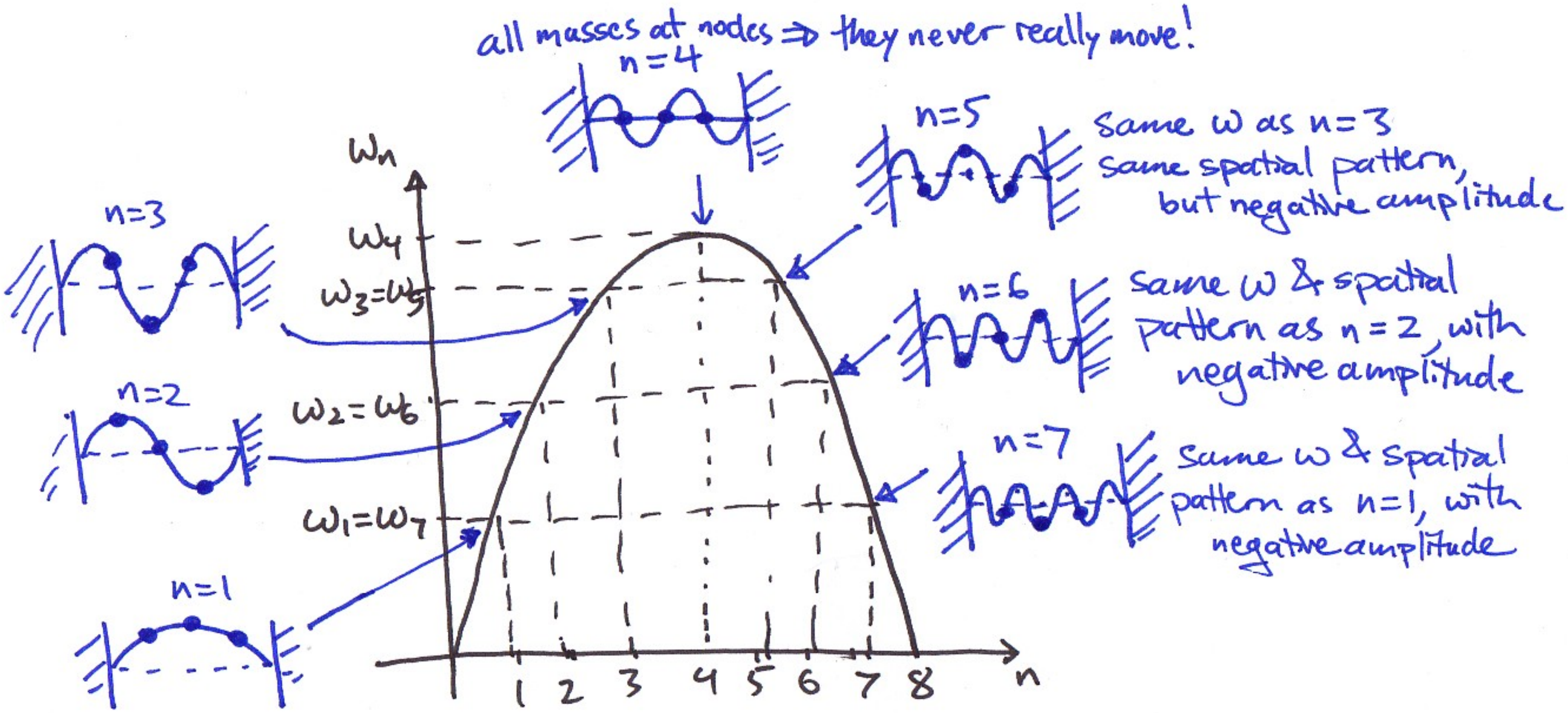
Summarizing:

$$\boxed{\begin{array}{l} y_j = A_n \sin k_n x_j \cos \omega_n t \\ x_j = ja \quad k_n = \frac{2\pi}{\lambda_n} = \frac{n\pi}{L} \quad \lambda_n = \frac{2L}{n} \\ n = 1, 2, \ldots \quad \omega_n = \sqrt{2}\,\omega_A \sin \frac{n\pi a}{2L} \\ \omega_A \equiv \sqrt{\frac{2T}{ma}} \end{array}}$$

Number of modes for the beaded string

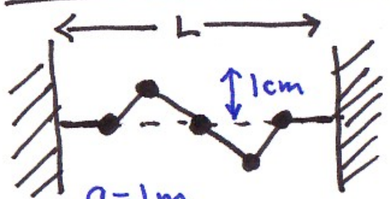
We know that the number of modes should equal the number of masses, yet there appears to be no limit on the mode index n.

$$\longrightarrow \omega_n = \sqrt{2}\,\omega_A \sin \frac{\pi}{2}\frac{n}{N+1}$$



⇒ the highest meaningful mode really is $n=N$.

An example of normal mode analysis for the beaded string



$a = 1\,m$
$m = 1\,kg$
$T = 1\,N/m$

All initial velocities $= 0$

Given this initial condition, what is

$$|y(t)\rangle = \begin{pmatrix} y_1(t) \\ y_2(t) \\ \vdots \end{pmatrix} ?$$

A) Express $|y(t)\rangle$ as a superposition of normal modes:

$$|y(t)\rangle = Re\left[\sum_{n=1}^{N} C_n e^{i\omega_n t} |e_n\rangle\right]$$

where $|e_n\rangle = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \end{pmatrix}$