

Physics 106b – 2011 Practice Problems for Final Exam

You should be able to do each of these problems (including all sub-parts) in about 15 minutes or less. If it takes you longer, you have not mastered the associated material thoroughly enough. **Note:** Of course, these problems don't cover exactly the same topics as the questions on the real exam. Therefore, doing well on these practice problems does not insure that you are adequately prepared for the exam. However, doing poorly on these problems does mean that you need to study more.

1. (12 points total): In a particular region of space, the electric field is given by $\vec{E} = x\alpha\hat{x}$, where $\alpha = 0.2 \text{ N/Cm}$.

a) (5 points) An electron is now fixed in place at $x = 5 \text{ m}$ and $y = z = 0$. What is the magnitude and direction of the electric force on the electron?

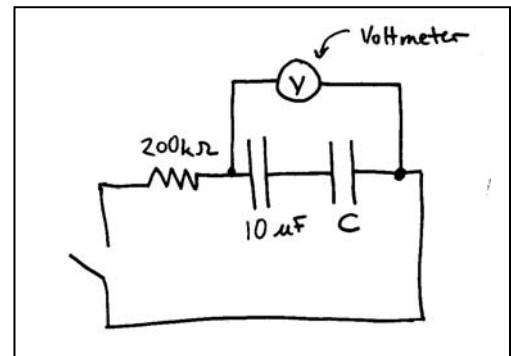
b) (7 points) With the electron still in place at $x = 5 \text{ m}$ (and the original electric field still acting) what is the voltage difference between the points $x = 10 \text{ m}$ and $x = 15 \text{ m}$? ($y = z = 0$ for both points)? Which is at the higher voltage? (*Hint: the voltage is produced by the combination of the original electric field and the electric field of the electron.*)

2. (12 points) A voltmeter is used to measure the voltage across a set of two capacitors, as shown here. Before the switch is closed, the meter measures 12 V. At $t = 0$, the switch is closed. At $t = 1.0 \text{ s}$, the meter measures 3.4 V. What is the capacitance C of the right capacitor?

Note: $200 \text{ k}\Omega = 200 \times 10^3 \Omega$

and $10 \mu\text{F} = 10 \times 10^{-6} \text{ F} = 1 \times 10^{-5} \text{ F}$

Answer: $6.57 \mu\text{F}$



3. (12 points total) Andrew and Peter are standing at either end of a log which is 50 m long. At a particular time, they simultaneously set off firecrackers. As they are doing this, a long train passes them, going at $V = 0.8 c$.

a) (5 points) How long is the log as measured by the observers on the train?

b) (7 points) How far apart are the firecracker explosions (*i.e.* what is the distance between the explosions), as measured by the observers on the train? (*Hint: the answer to this is different than the answer to part a.*)

4. (8 points total) Suppose we try to model an electron as a sphere of radius R whose charge is uniformly distributed over its surface:

a) (6 points) Calculate the energy stored in the electric field of the electron. *Hint: for spherically symmetric integrals, the infinitesimal volume element is $dV = 4\pi^2 dr$*

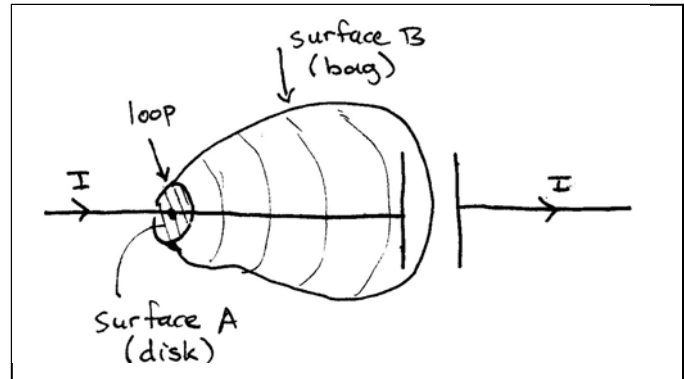
b) (2 points) Give an argument as to why a true “point” charge is an impossible idealization.

5. (12 points) You are called on to be an expert witness in a court case. One witness testified that he used his high magnification binoculars to read the license plate number of a car which was 10 km away. He shows the binoculars to the court. You estimate the diameter of the objective lenses to be about 5 cm. Make an argument as to why the witness must be lying. (Assume that the wavelength of light is about 500 nm.)

6. (12 points total)

a) (6 points) Explain why the original version of Ampère's law, $\oint \mathbf{B} \cdot d\vec{\ell} = \mu_0 I_{net \text{ threading}}$, cannot fully

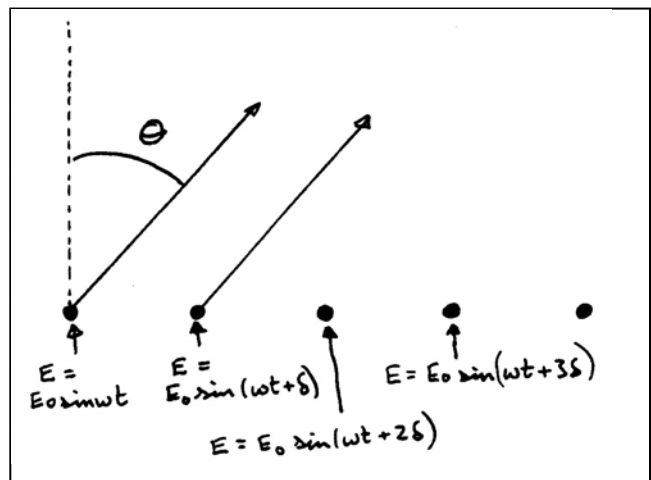
explain the situation shown here, in which a parallel plate capacitor is being charged by a current I . In your explanation, make use of the two open surfaces shown, both of which terminate on the loop shown.



b) (6 points) Using Gauss's law and $Q = CV$,

explain quantitatively why the corrected version of Ampère's law resolves the problem you described in part a.

7. (8 points) In RADAR ("Radio Detecting And Ranging"), a beam of radio waves is sent out in a particular direction. If it strikes a metallic object, part of the beam is reflected back. By measuring the time it takes for this echo to return, the radar unit can determine the distance to the object. In traditional radar, the beam is scanned across the sky by a rotating paraboloid dish antenna. However, it is often desirable to scan the beam more quickly than this allows. In "phase array radar", the beam is scanned across the sky electronically, by changing the voltage phase between elements of an array of antennas. The figure shows a cross-section of such an array.



Assume that the driving electronics are arranged so that the electric field at the surface of the leftmost antenna has magnitude $E = E_0 \sin(\omega t)$, while that at the surface of the next element to the right has magnitude $E = E_0 \sin(\omega t + \delta)$, that at the surface of the third element has magnitude $E = E_0 \sin(\omega t + 2\delta)$, etc. **In terms of d (the spacing between antenna elements), ω , and δ** , find the angles θ of the beam for which there is constructive interference between adjacent elements.

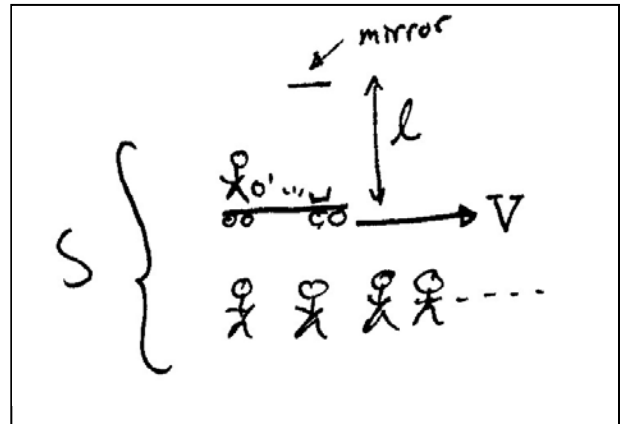
(As it turns out, most of the energy goes into the $m = 0$ beam. Now you can see that, by varying δ , which can be done electronically, the angle of the main beam can be quickly changed.)

8. (12 points) You are using an antenna consisting of a wire which has been formed into a circular loop or radius 15.0 cm to detect sinusoidal electromagnetic waves for which the amplitude of the electric field is $E_0 = 0.200$ V/m. If the wave frequency is 600 Hz, and if the loop is oriented for optimal reception, the amplitude of the emf induced in the loop is approximately

- a. 32.1 nV b. 84.3 nV c. 66.7 nV d. 178 nV e. 643 nV

Show your reasoning. *Hint: Start by giving a quantitative explanation of why it's reasonable to assume that, at any instant, the magnetic field is uniform over the area of the loop.*

9. (12 points) In this problem you will derive relativistic time dilation from the fact that c is the same in all frames. Consider the situation shown here. As seen in S , observer O' is moving to the right with velocity V . O' has a light clock with her. Show that the time between ticks of this clock as measured in S is given by $\Delta t = \gamma \Delta t'$, where $\Delta t'$ is the time between ticks as measured by O' and $\gamma \equiv \frac{1}{\sqrt{1 - V^2/c^2}}$.



In your derivation, you may only use the following:
 speed = distance/time (when all three are measured in the same reference frame)
 speed of light is constant in all reference frames and is equal to c
 Pythagorean theorem
 working principles of light clocks
 common sense
 algebra

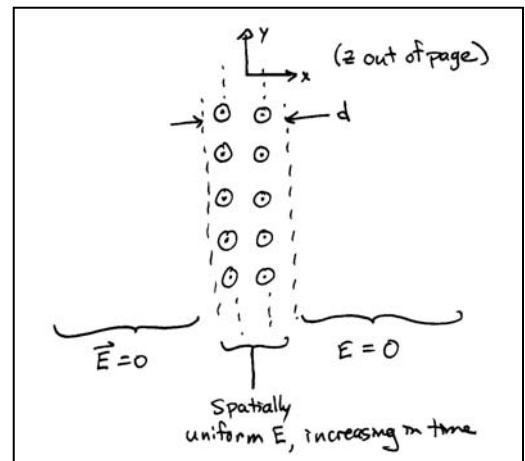
10. This problem begins with the same setup as one of the problems for exam 2.

That the electric field in this problem is of the following form throughout a region of vacuum:

$$\mathbf{E} = \beta t \hat{\mathbf{z}} \quad \text{for } -\frac{d}{2} < x < \frac{d}{2}$$

$$\mathbf{E} = 0 \quad \text{for } x < -\frac{d}{2} \quad \text{and for } x > \frac{d}{2}$$

In the above, t is time, and β is a known positive constant. Another way of expressing the above is that the magnitude of \mathbf{E} increases at the constant rate β , and that \mathbf{E} is only non-zero inside an “infinite slab” of thickness d . (As shown by the first equation above, the region of non-zero \mathbf{E} is infinite in the y and z directions. Within this region, \mathbf{E} is spatially uniform, and increasing in time.) There are no currents anywhere that you need to worry about.



Using a loop in the plane of the page, you showed that, for $x > 0$ the magnetic field points in the y -direction, and is given by $B = \begin{cases} \mu_0 \epsilon_0 \beta x & \text{for } 0 < x < d/2 \\ \mu_0 \epsilon_0 \beta d / 2 & \text{for } x > d/2 \end{cases}$. Now, use a rectangular loop perpendicular

to the page, with the left arm of the loop at $x = 0$ and the right arm of the loop at $x = 1.1 d/2$, so that the right arm of the loop is slightly outside the region of non-zero \mathbf{E} . Is this combination of electric and magnetic fields consistent with Faraday’s law for this loop?