

Physics 106b – 2011 Practice Problems for Exam 2

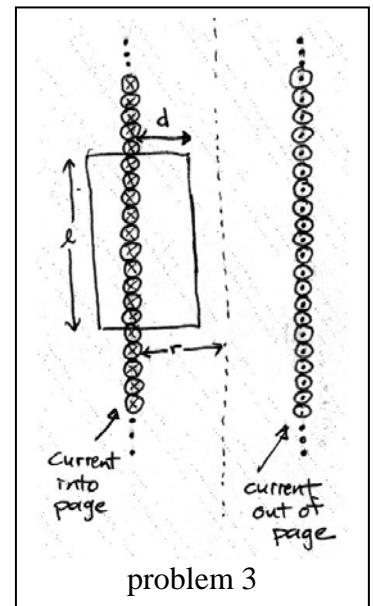
You should be able to do each of these problems (including all sub-parts) in about 15 minutes or less. If it takes you longer, you have not mastered the associated material thoroughly enough. **Note:** Of course, these problems don't cover exactly the same topics as the questions on the real exam. Therefore, doing well on these practice problems does not insure that you are adequately prepared for the exam. However, doing poorly on these problems does mean that you need to study more.

1. (10 points) Explain why the following combination of electric and magnetic fields violates at least one of Maxwell's equations: $\mathbf{E}(x, y, z, t) = 0$ $\mathbf{B}(x, y, z, t) = B_0 \frac{z}{r_0} \hat{\mathbf{z}}$ where $B_0 = 0.01$ T and $r_0 = 10$ m.

(Note: you need only explain why it violates one equation, even though it might violate more than one. Also, as is appropriate for this exam, you should use the original version of Ampère's law, which is correct for a constant electric field such as this one.)

2. (15 points) A point charge of mass m and charge q is moving in a circle in a uniform magnetic field B . Starting from the equation for the Lorentz force (i.e. the magnetic force on a moving point charge), find the period T of this motion. *Hint: you should find that the period is, perhaps surprisingly, independent of the velocity v of the point charge and also of the radius r of the circle!* Also, recall that the centripetal force is given by $F = \frac{mv^2}{r}$.

3. (15 points) Part of a very long, straight solenoid is shown in cross-section. Apply Ampère's law to the loop shown to find the strength of the magnetic field inside the solenoid in terms of the current I (constant in time) and n , the number of turns of wire per meter (along the vertical direction). You should find that B doesn't depend on the distance d from the wall of the solenoid or on the radius r of the solenoid. Because of the symmetry of the situation, you may assume that the field inside the solenoid is parallel to the axis of the solenoid. Because the solenoid is very long, you may assume B is zero on the part of the loop that is outside the solenoid.



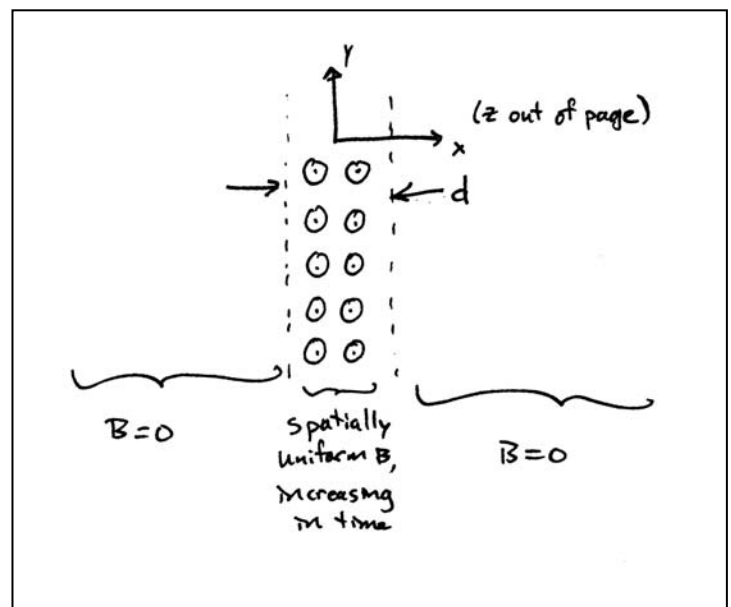
4. (15 points) A piece of superconducting wire ($R = 0$) is connected to a battery bought at Radio Shack. The formula $P = V^2/R$ seems to indicate that the power dissipated in the superconductor is infinite, yet $P = I^2 R$ seems to indicate that no power is dissipated. Resolve this dilemma.

5. (20 points total) For this problem, the magnetic field is of the following form throughout all space:

$$\mathbf{B} = \beta t \hat{\mathbf{z}} \quad \text{for } -\frac{d}{2} < x < \frac{d}{2}$$

$$\mathbf{B} = 0 \quad \text{for } x < -\frac{d}{2} \text{ and for } x > \frac{d}{2}$$

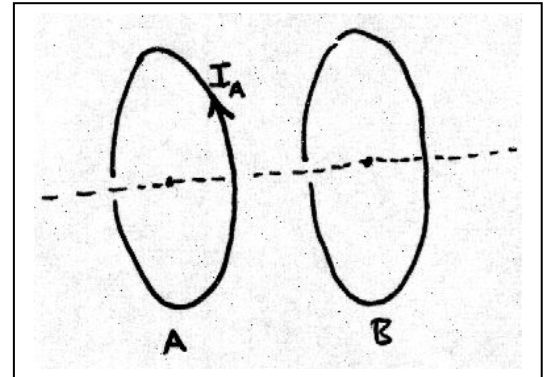
In the above, t is time, and β is a known constant. Another way of expressing the above is that the magnitude of \mathbf{B} increases at the constant rate β , and that \mathbf{B} is only non-zero inside an "infinite slab" of thickness d . (As shown by the first equation above, the region of non-zero \mathbf{B} is infinite in the y and z directions. Within this region, \mathbf{B} is spatially uniform, and increasing in time.) There are no static electric charges present.



a. (15 points) What is $\mathbf{E}(x)$ for $x > d/2$ (i.e. outside the “slab”) ? Be sure to specify both magnitude and direction, and show your reasoning clearly.

b. (5 points) What is $\mathbf{E}(x)$ for $0 < x < d/2$ (i.e. inside the “slab”) ? Be sure to specify both magnitude and direction.

6. (20 points) The two circular loops of wire in the figure have their planes parallel to each other. As viewed from the left (i.e. from A toward B), there is a counterclockwise current in loop A, and this current is increasing. Give the direction of the induced current in loop B, and state whether the loops attract or repel each other. Explain your reasoning briefly.

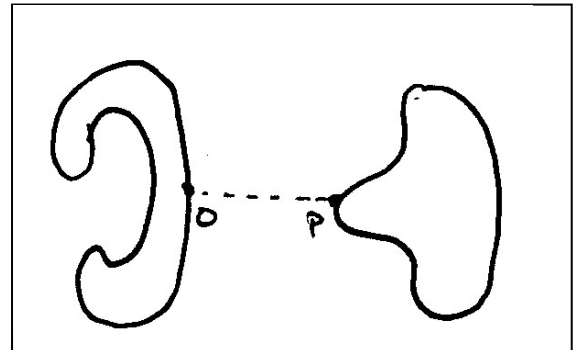


7. (10 points total) For both parts, assume \mathbf{E} is constant in time.

a. (3 points) Give an example of a situation for which it you could use Ampere’s law to find the magnetic field. (You need not actually do the calculation, but rather just describe the situation.)

b. (7 points) Give an example of a situation for which it would not be possible to use Ampere’s law to find the magnetic field. How could you, in principle, find the magnetic field for this situation? (You need not actually do the calculation.)

8. (25 points) Two irregularly-shaped conductors are shown here. A charge $+q$ is applied to the left conductor, and $-q$ to the right conductor. Point P is a distance x_o directly to the right of point O ; both points are on the surface of the conductors. Taking point O to be the origin, the electric field at points along the straight line connecting O to P is now found to be $\mathbf{E} = (Aq + Bqx)\hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ points to the right and A and B are constants. What is the capacitance between the two conductors? (Hint: start by finding the voltage difference between the conductors.)



9. (10 points) Consider an isolated balloon. Assume the balloon is spherical and that it carries a charge Q uniformly distributed on the surface. (Further assume that κ (air) = 1, i.e. ignore the tiny reduction in the electric field caused by the polarization of the air molecules.) If the balloon expands from a radius r_o to a new radius $(r_o + \Delta r)$, what is its change in potential energy? (Hint: use the energy density of the electric field.)