

Energy Eigenstates. Words by Walter F. Smith, 4-22-08

Tune: "A Comedy Tonight", from "A Funny Thing Happened on the Way to the Forum",
by Stephen Sondheim

1. Take a wavefunction, any wavefunction –
You can synthesize it out of energy eigenstates!
Why, might you ask, should I do this task?
Relax, and I will sing to you of energy eigenstates!
No thinking hard! Let down your guard!
Once I have sang of it, you'll get the hang of it!
Stop feeling queasy – it's really easy.
I'll tell you all about their traits!
...My favorite functions, energy eigenstates!

2. To be a bit formal, they're orthonormal.
That means the inner product of two energy eigenstates
Is zero or one, and – here comes the fun- --
The secret to expand-ing in energy eigenstates!

$$\langle \psi_m | \psi_n \rangle = \delta_{mn}$$

$$\psi = \sum_n c_n \psi_n$$

All we could wish! The coefficient in the sigma is no enigma:
c-n is psi-n star times your psi, then
Integrate it over all of space!

$$c_n = \int_{-\infty}^{\infty} \psi_n^* \psi dx = \langle \psi_n | \psi \rangle$$

It's the inner product with the energy eigenstate!

3. They're not dramatic, in fact they're quite static.
Nothing ever changes in an energy eigenstate!

$$\frac{\partial \langle A \rangle}{\partial t} = 0 \text{ for any observable}$$

for an energy eigenstate

Minus $i E_n t$, over \hbar we see
Is the exponential for an energy eigenstate!
Stately and calm, they've no need for haste.
The lords and ladies of Hilbert space-!
But add them up right now, you'll see a sight now,
With the time dependence it creates!

$$\Psi_n(x, t) = \psi_n(x) e^{-iE_n t / \hbar}$$

Levels electronic! Spherical harmonics!
Radial equation! Harmonic oscillation!
Tunneling authentic! Atoms hydrogenic!
Paraboloid, trapezoid, sinusoid, overjoyed!

Stately and calm, they've no need for haste.
The lords and ladies of Hilbert space-!
But add them up right now, you'll see a sight now,
With the time dependence it creates!
Superpositions of energy eigenstates!