

# ABSTRACT

## EVOLUTION AND INTERNAL DYNAMICS OF QUASI-STATICALLY SHEARED GRANULAR FLOWS

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Granular materials are collections of particles whose sizes range from microns to meters. They can be packed as a static solid pile and can also move like fluid or gas. When driven at a sufficiently low rate under a compressive load, the grains flow in a quasi-static regime, in which they creep while maintaining simultaneous contacts with multiple neighbors.

We investigate experimentally a quasi-static flow of glass beads packed and sheared in an annular channel. The experiments utilize techniques of refractive-index-matched fluorescent imaging, particle tracking, and simultaneous measurements of volume and boundary shear force. Under long-term shearing, a crystallization transition accompanied by a step-wise decrease of packing volume and shear force can occur. This transition also alters the structure of the internal velocity field. Boundary conditions can affect the crystalline ordering throughout the entire packing. We find that, even under identical boundary conditions and shearing, the evolution of the packing can lead to non-unique final states. The behavior in response to shearing is influenced by the past history of the packing.

Our measurements of the internal velocity fields have a dynamical range of five decades; parameters such as packing thickness and particle size are varied systematically. We demonstrate that crystalline ordering has a significant impact on the spatial gradient of grain velocity. Changing particle size does not influence the gradient of particle velocity significantly; the velocity decay length does not show a direct scaling with particle size. We also make time-resolved measurements of the internal flows in response to cyclic shearing,

and investigate the anomalous motility of individual grains in response to the reversal of boundary motion. Our observations of both steady and transient states illustrate that sufficient packing thickness is needed to reveal bulk properties, such as the shear banding of velocity field, the development of distinct states of internal order, and the anomalous mobility upon shear reversal. In addition, by analyzing the measurements in this and other experimental systems of granular shear flows, we argue that the spatial decay of grain velocity should be geometry-specific; a heuristic model is proposed to explain the shear banding in this geometry.

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