the typical local displacement accumulated during the transient, as displayed at the right axis of Fig. 5.13(b), shows a different steepness than the decay of the steady-state mean velocity with observation height (in this case $-H_0/2$).

It would be very interesting to construct a minimal theory to predict the development of this shear-reversal transient as the layer thickness is increased, as well as the duration of the transient (about $10^1 d$ of boundary displacement).

5.3 Discussion

5.3.1 Shear banding

In this section, I discuss on the origin of shear banding, i.e. the localization of velocity to a narrow region (the so-called shear band), and the factors that affect the steepness of the decay of particle velocity.

The sheared granular packing studied in our experiments exhibits shear banding at the upper driving boundary, accompanied by a continuously decaying velocity profile with a finite slip at the stationary bottom; see Fig. 5.3, Fig. 5.4, and Fig. 5.5. Though the internal ordering of grains alter the transverse velocity profiles at each horizontal plane, the global trend of shear banding with respect to height is shared by both the ordered and the disordered states of flow. Shear banding is commonly observed in both experiments and simulations of granular flows: the shear band more often occurs in the vicinity of a driving boundary [36, 5] or under a free surface [27]; it can also be found in the deep interior of a granular flow [16], depending on the global geometry of a model system and other parameters such as the stress level [1]. Recently, a theoretical study [58] proposes a way to determine the location of the shear band in the situation described in Ref. [16], by treating it as a variational problem that minimizes the frictional dissipation. In this theory, the shear band is treated as a mathematical surface whose thickness is infinitesimal.

From a purely theoretical point of view, a two-dimensional (2D) packing with no gravity, sheared by two identical parallel boundaries moving steadily relative to each other, has reflection symmetry with respect to the mid-plane between the two boundaries. If a theory would predict a unique steady-state profile for this 2D problem, the predicted profile must have the same reflection symmetry. A linear velocity profile is one possibility (although not the only one), while a one-sided shear band clearly does not satisfy the reflection symmetry. For instance, the simulated steady-state velocity field of a thick 2D granular shear flow as reported in Ref. [59] would approach a linear profile [57], if the conditions are set to be symmetric by turning off the gravity (which was non-zero and pointed towards one of the shearing boundaries in the situation reported in Ref. [59]). The alternative theoretical scenario for an asymmetrical steady-state profile in a symmetrical 2D problem is to allow non-unique stationary velocity profiles that are selected by prior history (initial condition) of the flow - see the simulated velocity profiles reported in Ref. [1]. Theories of this type would need to contain internal state variable(s) other than the velocity gradient in order to register the effect of prior history on the evolution of the velocity field. Beyond the idealized 2D scenarios described above, the commonly observed asymmetrical shear banding is totally reasonable if the effects of sidewalls are included using a full 3D theory. The author is not aware of such a 3D theory that can be directly applied to our sheared packings in the annular channel.

Our experiments of sheared granular packing do not satisfy the reflection symmetry of a simple 2D problem. In the following discussion, I examine four factors that break the reflection symmetry: (1) the weight of individual grains under the influence of gravity, (2) drag force from the interstitial fluid, (3) the initial state at which all particle are at rest, and (4) the stationary sidewalls attached to the bottom. The grain weight and fluid drag (1,2) are insignificant factors, since it can be estimated that our large normal load creates inter-particle compression forces that are a few orders of magnitude larger than individual grain weight and the maximal viscous drag by the fluid. In Hanes and Inman's experiments [19] using an annular shear channel comparable to our setup except with their upper boundary being at rest, the shear band always occurs near the upper stationary boundary. This observation suggests that the initial velocity distribution of grains alone is not sufficient for creating a 'preference' for nearly zero velocity in the bulk and a shear band located at a mobile boundary. The velocity field measured across the channel at a constant height (Fig. 5.1) shows a significant drag by the sidewalls. The analyses above suggest that the stationary sidewall (4) seems to be the dominant [1] factor in determining

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[1] Although the initial velocity distribution of grains may not affect the trend of shear banding, the initial state of packing may influence the evolution of the packing and the selection between different states of

the general trend of the vertical profile (shear banding). In Section 5.3.5, I use a heuristic model to explain that shear banding can emerge as a consequence of resistance exerted by the sidewalls.

5.3.2 Characteristic decay length of velocity field

The steepness of the spatial decay of velocity field in a shear flow is often characterized by an exponential decay length, sometimes called the width of the shear band. The characteristic length is influenced by both the properties of grains and geometrical factors of the system. First of all, we demonstrate in Fig. 5.3 that the state of internal order decisively affects the steepness of the velocity decay. The decay lengths derived from the master curve for sufficiently thick packings (Fig. 5.4) are apparently consistent with the conventional wisdom that the width of a shear band at the vicinity of the driving boundary is comparable to a few particle diameters. However, Fig. 5.5 shows that the velocity profiles of different sizes of particles clearly do not fall on the same curve when the distance from the shearing surface is scaled in units of particle diameters. Rather, the trends of velocity decay for different bead sizes seem similar in terms of actual physical distance. These observations suggest that the spatial decay rate of particle velocity is controlled primarily by the channel geometry, while the exact steepness of the decay can be fine-tuned by the internal structure of the sheared packing.

The experimental evidence that we and other researchers have reported suggest that the spatial decay of velocity as a function of distance from a driving surface in a boundary-driven granular shear flow can be geometry-specific and strongly influenced by the interaction with the stationary walls that confine a granular packing. In fact, there are no obvious theoretical reasons explaining why the decay length should depend explicitly on the grain size. It may not be surprising that the measured decay curves in different experimental granular flows not only show a large dispersion of values in units of grain diameter, but also show differences in functional forms.

For instance, the velocity decay lengths in units of grain diameter obtained from the interior of a thin Coutte-cell flow driven by its inner cylinder [36] are between 2 and 1, lower than the typical values extracted from our velocity fields near the driving boundary.

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order(explained in Chapter 4); therefore the initial state can still affect the exact velocity field indirectly.

Nasuno and coworkers [27] determined the spatial decay of the creeping velocity far below a inclined surface of flowing grains, by monitoring the grain motion visible through the transparent sidewall: the vertical profile they obtained shows a functional form that is almost purely exponential, quite distinct from the velocity profiles reported in Ref. [36] and in our work. But note that the grain motion measured in Ref. [27] are under the direct influence of the sidewall, as opposed to the internal measurements in our work and Ref. [36]. This factor may be responsible for distinct functional forms of velocity decay.

Would a shear flow experiment approach a quasi-2D problem as the cross-sectional aspect ratio goes to extreme? In our system, when the packing thickness H_0 is small compared to the fixed channel width W_0, i.e. $(H_0/W_0) \to 0^+$, the flow can be considered as a quasi-2D limit, but is perhaps a trivial one. The velocity field of a 5-layer in Fig. 5.4 exhibits a nearly linear vertical profile. Note that this linear flow limit does not reflect anything about the sheared material - an ordinary fluid in a the same channel with a small H_0/W_0 would behave the same way. On the other hand, the system is not designed in a way that W_0 can be increased independently to test the limit. On the other hand, the standard Coutte-cell experiment with a rotating inner cylinder (in the vertical direction) is commonly believed to be close to a quasi-2D problem when the total filling height of the material (L_F) is large compared to the gap width (ΔR). But note that the asymmetry caused by the open upper surface, the vertical gradient of compressive stress inside the material, and the bottom condition can have significant effects on the flow. (Ref. [16] illustrates the long-range influence of the conditions on the bottom.) It is relatively inconvenient to vary ΔR continuously to study the possible influence of packing thickness on the velocity gradient, compared to our system (as shown in Fig. 5.4). More importantly, the commonly observed fact that granular Coutte flows with an extremely small aspect ratio ($\Delta R/L_F$) still exhibit shear banding as opposed a symmetric linear profile can involve an intrinsic asymmetry in this geometry: the internal shear stress ($\sigma_{\phi r}$) varies as $\sigma_{\phi r} \propto 1/r^2$ as a natural consequence of the conservation of angular momentum. This variation of stress may play an important role in the spatial decay of velocity, particularly for quasi-static granular flows, as we discuss further in Section 5.3.5.

To test theoretical ideas that may integrate the experimental information of dense granular shear flows up to date, the analyses in this subsection indicate that implementing a theory in a 3D context is necessary. A decisive test of theories would be to see whether incorporating the interaction of the sidewall in various experimental geometries can produce the observed general trend of the velocity field (shear banding). [2]

In the preceding discussions, the curvature of the circular channel in our system and other Coutte-cell experiments are generally ignored. Circular geometries are advantageous for eliminating "end effects" and are commonly used in both 2D and 3D experiments. But in view of the long-range "force chains" (as can be visualized by the use of photo-elastic materials as in Ref. [24]), the finite radius of curvature may still play a minor role in the flow, especially for quasi-statically packed grains.

5.3.3 Development of bulk properties with packing thickness

Several notable bulk properties observed in our system occur only if the packing thickness is sufficient:

Distinct stats of order – In the experiments of our typical particles with $d = 0.68$mm, we find that about 15 layers of materials or more are needed for developing two clearly distinguishable states of order. The states of internal order for experiments with less amount of glass beads depend on the exact condition of the bottom surface: the degree of order can be highly sensitive to layer thickness (Fig.3.4), or can be inhomogenous around the channel (with the shear force close to the typical value for a disordered state. For packings thicker than 30 layers, spatially non-uniform states with a detectable vertical gradient of local order can be created and persist indefinitely. (Please see Section 3.2, Section 3.3, and Fig. 5.3 for these observation.) Therefore, the "available phase space" of the system in terms of its state of order depends on system size; our typical packing size (24 layers with W_0/d= 28.5) lies within the range that a simple two-state scenario is sufficient.

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[2] In units of particle diameter, the spatial decay of velocity field produced in existing 2D theories and simulations such as Ref. [59, 1] generally have a length scale an order of magnitude larger than all experimental measurements reviewed here.