state and 0.26 for the crystallized state -see Fig. 3.1 for the change at the crystallization transition. These values are close to the typical values reported in the simulation of Ref. [1], but significantly smaller than the friction coefficients shown simulations reported in Ref. [22] and Ref.[59]. The discrepancy may be partly due to different settings of material parameters. Also, by comparing the measured shear force between dry-particle and immersed-particle experiments, we find that the presence of the immersion fluid reduces the macroscopic shear force by about 12 percent in our experiments, presumably due to lubrication.

5.2 Transient response to shear reversals

To obtain deeper insight into the quasi-static grain dynamics, we extend our investigation from steady states to transient responses. Here we focus on a particular type of transient — the brief period during which a sheared packing readjusts its internal structure to a new steady state after the direction of shearing has been reversed. During this period, grains appear anomalously mobile compared to their steady-state behavior. We refer to this behavior as a shear-reversal transient, which may be regarded as a brief period that the material is "soft". The investigation into the transient behaviors of dense granular flows may provide useful information for going beyond a mean-field description.

5.2.1 Grain motions during the shear-reversal transient of a thick packing

For either the disordered or the crystallized state, one can stop the steady shearing and then resume the boundary motion in the opposite direction. The reversal of shear causes extraordinary motions of internal grains, and successive sinking of the driving boundary under a fixed normal load. In this section, we use a typical packing thickness (200g of glass beads of diameter d =0.68mm, which would form a 24-layer structure when crystallized) as an example to illustrate the generic behavior of the shear-reversal transient.

Fig. 5.8 demonstrates the displacements of individual grains sampled on an internal vertical (xz) plane of a disordered state, whose boundary motion is being switched from +x to -x. During the first 12d of travel of the boundary motion in the -x direction (b), the particles move significantly more, compared to the displacements during the same amount of boundary motion in the prior steady state (a). However, unlike the behavior of an avalanche, all grains are still slower than the speed of the driving boundary. The displacements during the next $12d$ of boundary motion in -x direction (c), become closer to the amounts shown in (a). The amplitude of response decays with height, but remains detectable down to the bottom of these images. By inspecting the sequential images frame-by-frame at a time interval corresponding to $0.1d$ of boundary displacement, we find no detectable propagation delay through the entire image, Furthermore, at any stage of the process including the middle of this transient, when the boundary motion is suddenly stopped, all grains stop essentially simultaneously within the same brief time interval. The lack of detectable propagation time for either the start of the stop of the grain motion suggests that the transient behavior is still a quasi-static behavior, where the role of grain inertia, if any, is negligible. For convenience, we call the behavior during this first $12d$ of boundary translation in the reversed direction a "soft period" of the flow.

Fig. 5.9 shows another measurement performed with a crystallized state. With repetitive reversals of boundary shearing, we compute the average velocity of about 60-80 grains on several horizontal planes as functions of time; the graph shows that the velocity peaks are nearly simultaneous throughout the entire packing and are well synchronized to the reversals of the shearing. The amplitudes of the peaks decrease with height, in a manner that is consistent with Fig. 5.8. The graph also includes an additional test confirming that the velocity peak does not occur if the boundary driving is resumed in the same direction. As the vertical distance from the shearing surface increases, the amplitudes of the transient peaks gradually decrease to the noise level of about 10^{-2}.(The noise level of 10^{-2} in Fig. 5.9 is consistent with a simple estimate obtained from the uncertainty in particle positions and the number of particles being averaged.)

The larger stochastic noise visible at $z = -7.5d$ in Fig. 5.9 is not measurement noise, but rather a manifestation of the collective intermittency illustrated in Fig. 5.2. Furthermore, inspecting the data of similar experiments with higher time resolution also reveals that the observed temporal width of the shear-reversal transient does not depend significantly on the height of measurement.

We also vary the driving speeds by two decades, with the same protocol of shear reversal as shown in Fig. 5.10. The time step in all three graphs corresponds to the time required for the boundary to translate a distance $0.096d$. Despite of two decades of change in the actual time scale, the peak widths (measured in terms of boundary displacement rather than time) are essentially the same for different driving speeds. Therefore,*the rate-independence is true not only for the steady-state velocity field* (as shown in Sec. 5.1.3) *but also for the time-dependent transient displacement during flow reversal.*

In our shear-reversal experiments, the driving protocol features a static period (typically 1s or 10s) during which the boundary is completely at rest; the shear stress is abruptly relaxed to zero near the end of this static period - there is a short period (typically 0.1s or 1s) during which the shear stress is essentially zero. We find that the grains start to move significantly at the moment when the motion of the boundary begins in the opposite direction, rather than at the moment of the release of shear stress. This observation suggests that the major effect reported here is not an energy exchange with a conservative energy storage mechanism such as gravity or the elastic deformation of grains. Instead, the extra mobility during the shear-reversal transient reflects the adjustment of the internal texture in order to adapt to a new direction of shear. The adjustment involves a gradual quasi-static reconfiguration of the contact network.

It is understandable that a sufficient amount of total boundary displacement is required for the packing to fully adapt to a direction of shearing. In Fig. 5.11, we demonstrate that, if the shearing in one direction is not sufficiently long before the next reversal, the next reversal transient can be significantly smaller, reflecting the reduced development of internal texture during the shearing in the previous direction. In general, we find that a total boundary translation of 10^2d is needed for the full development of the packing in each shearing direction, in the sense that the grains then respond to the shear reversal with an amplitude and shape that is insensitive to the shearing accumulated previously. (All shear-reversal curves shown in this section are measured under the condition that the boundary has continuously traveled in one direction for at least $400d$ prior to the point defined as $t = 0$ in each graph.) *We can probably regard this* 10^2d *of unidirectional shearing as a process of "anisotropic hardening" towards a specific direction. The "soft period" can be also regarded as the transition from one "hardened" state to the other.*

The obvious difference between the curves of internal grain velocity $V_x(t)$ and that of driving velocity $U(t)$, as shown in Fig. 5.9 and other figures reflects a complicated non-linear relation between the local average displacement and the displacement of the driving boundary. Unlike the case in conventional non-linear visco-elasticity where there is a small amplitude below which a linear description can be a starting approximation of the material response, no linear regime can be defined in describing this important "soft period", that reflects the reconfiguration of the internal network to accommodate the new direction of shearing. Clearly, to go beyond a mean-field description of the grain motion inside dense granular materials, the development of more advanced theoretical tools that do not require a linear regime as the starting approximation are necessary.

5.2.2 Change of volume in response to shear reversals

Repetitive reversals of shearing result in significant sinking of the upper boundary that is driven under a fixed normal load. Fig. 5.12 shows the vertical displacement of the upper boundary measured once during each reversal of its direction of motion. The successive sinking occurs in both crystallized and disordered states; in fact, the total sinking caused by multiple reversals of shearing can be more than the amount caused by the change of bulk density due the structural change in the interior. For either the crystallized or disordered state that has been sufficiently compacted by a long-term shearing (driving in one direction for more than $10^5 d$), applying the repetitive shear reversals (oscillatory shearing) does not change the internal order significantly; the packing tends to gradually recover to the previous stationary volume when the unidirectional shearing is resumed, shown as the data following $t = 14850$s or $t - t_0 = 1000$s. These facts suggest that the volume compaction induced by shear reversal is highly non-uniform, and primarily localized in the upper layers. Furthermore, when the upper boundary starts to move in a reversed direction, its height makes a sudden descent of about $d/5$, followed by the much more gradual recovery described above. Although the sudden descent is not shown in Fig. 5.12 in which the data are collected discretely (once every revolution of the rotating boundary), this phenomena can be demonstrated by the continuous sampling of the boundary height as the shearing reverses. These time-resolved measurements of the change of boundary height can perhaps be used to test theories or computer simulations of packed grains in a rather direct way.

5.2.3 Development of shear-reversal transient with packing thickness

We find that the occurrence of the shear-reversal transient, which may be coined as a "soft period", depends on having a packing that is sufficiently thick. In this section we report the internal response to shear reversal at different packing thickness H_0. In Section 5.2.1, we have established that the anomalous mobility occurs throughout the entire packing with negligible propagation delay when the motion of the upper boundary is reversed. In the experiments shown in Fig. 5.13, we use the time-resolved velocity measurement at the mid-height as the indicator of the occurrence of the anomalous mobility. The top row of Fig. 5.13 shows the protocol of boundary driving that is shared by experiments of different packing thickness. The anomalous mobility becomes identifiable as the packing thickness increases to 13 layers or larger. These experiments are performed using the typical particles with $d = 0.68$mm. The characteristic duration of the anomalous motility, when it occurs, is insensitive to the packing thickness. The typical duration is labeled as τ on the graph and corresponds to about $10d$ of the translation of the driving boundary. On the other hand, by using progressively larger particles with the vertical dimension kept fixed as 15 layers, we find that the anomalous mobility become less significant and eventually undetectable when the particle diameter d is as large as 2mm. Therefore, sufficient numbers of layers in both vertical and horizontal dimensions (H_0/d and W_0/d) seem essential for the phenomena of this "soft period" to occur.

In Fig. 5.4(b), we quantify the contrast between the transient stage and the post-transient stage by time-averaging the velocity; the deviation of the two values from each other marks the characteristic layer thickness above which the shear-reversal transient is observed. The average value of the post-transient velocity serves as an estimate of the true asymptotic steady state velocity at the mid-height ($z = -H_0/2$) for different packings; at large thicknesses, this value reflects the decay with z that is consistent with the master curve [Fig. 5.4] of the steady-state velocity profile for a sufficiently thick packing. (Compared to Fig. 5.4, the larger error bar here and its non-smooth variation with z is due to insufficient total sampling time– a compromise for obtaining the required time-resolution.) In addition, the typical local displacement accumulated during the transient, as displayed at the right axis of Fig. 5.13(b), shows a different steepness than the decay of the steady-state mean velocity with observation height (in this case $-H_0/2$).

It would be very interesting to construct a minimal theory to predict the development of this shear-reversal transient as the layer thickness is increased, as well as the duration of the transient (about $10^1 d$ of boundary displacement).

5.3 Discussion

5.3.1 Shear banding

In this section, I discuss on the origin of shear banding, i.e. the localization of velocity to a narrow region (the so-called shear band), and the factors that affect the steepness of the decay of particle velocity.

The sheared granular packing studied in our experiments exhibits shear banding at the upper driving boundary, accompanied by a continuously decaying velocity profile with a finite slip at the stationary bottom; see Fig. 5.3, Fig. 5.4, and Fig. 5.5. Though the internal ordering of grains alter the transverse velocity profiles at each horizontal plane, the global trend of shear banding with respect to height is shared by both the ordered and the disordered states of flow. Shear banding is commonly observed in both experiments and simulations of granular flows: the shear band more often occurs in the vicinity of a driving boundary [36, 5] or under a free surface [27]; it can also be found in the deep interior of a granular flow [16], depending on the global geometry of a model system and other parameters such as the stress level [1]. Recently, a theoretical study [58] proposes a way to determine the location of the shear band in the situation described in Ref. [16], by treating it as a variational problem that minimizes the frictional dissipation. In this theory, the shear band is treated as a mathematical surface whose thickness is infinitesimal.

From a purely theoretical point of view, a two-dimensional (2D) packing with no gravity, sheared by two identical parallel boundaries moving steadily relative to each other, has reflection symmetry with respect to the mid-plane between the two boundaries. If a theory would predict a unique steady-state profile for this 2D problem, the predicted profile must have the same reflection symmetry. A linear velocity profile is one possibility (although