Assignment 9
Due: Friday, Nov. 16 at 4 pm
Reading: Livingston 13.1-13.3

Assigned exercises (group problems unless otherwise noted):

Livingston:
12-13
12-14

9A. Individual problem. Using a diagram, make a clearer explanation of the second paragraph on Livingston p. 229 (“The contact potential is…”)

9B. Derivation of the Richardson-Dushman equation for thermionic emission. In this problem, you will derive the expression for the current density emerging from the face of a rectangular sample, with face area *A* parallel to the *x*-*y* plane and thickness *L* along the *z*-axis. (We’ll call the positive *z* direction “up”.)

a. Explain why the current emitted from the top face of this sample is given by $I = -\frac{e}{L}\sum v_{z,\mathbf{k}}$, where $v_{z,\mathbf{k}}$ is the *z*-component of the velocity for a state with wavevector **k**, and the summation runs over all states that are 1) occupied, 2) have enough energy to escape from the sample, and 3) have a wavevector **k** with a positive *z*-component.

In order to complete the above sum, we’d need detailed knowledge about the states within the solid sample. However, we can simplify the calculation tremendously, and also make it completely general, with a simple assumption. We’ll assume that the electrons in the sample are in thermodynamic equilibrium with a gas of free electrons in the space above the sample. This means that the electrons in the gas must have the same distribution of energies as does the population of the electrons in the sample that have high enough energy to escape. Therefore, we can sum over the states in the gas (which are simple free electron states), rather than the more complicated states in the sample. We will use the usual convention that $PE \equiv 0$ infinitely far from the sample, so that the total energy of an electron in the gas is simply $\varepsilon = \frac{\hbar^2 k^2}{2m}$.

b. Using the results of Periodic Boundary Conditions for the *k*-space volume per allowed state, explain why the current density can then be written

$$J = -e \int_{k_z > 0} \frac{1}{4\pi^3} v_{z,\mathbf{k}}\, F\left(\varepsilon = \frac{\hbar^2 k^2}{2m}\right) d^3\mathbf{k}\,,$$

where F is the Fermi function, which is a function of the energy $\varepsilon = \frac{\hbar^2 k^2}{2m}$; it is therefore written as $F\left(\varepsilon = \frac{\hbar^2 k^2}{2m}\right)$ inside the integral. (Note: the limit of integration comes simply from the fact that, given our choice for where $PE \equiv 0$, any state with positive energy has enough energy to escape, but that it needs an upward pointing wavevector to escape the top surface of the sample rather than the bottom. You need not discuss the limit on integration further in your answer.)

Problem continues on the next page

c. Explain why the factor $\varepsilon - \varepsilon_F$ which appears in the Fermi function can be written as $\varepsilon - \varepsilon_F = \dfrac{\hbar^2 k^2}{2m} + \phi$, where ϕ is the work function. Hint: you will find it helpful to draw an energy level diagram.

d. As discussed in class, the states that are capable of escape are far into the "Maxwell-Boltzmann tail" of the Fermi-Dirac distribution, so you should use the appropriate approximate form for the Fermi function. Use $v_{z,\mathbf{k}} = \dfrac{\hbar k_z}{m}$ to show that the current density is given by the Richardson-Dushman equation,

$$J = -\frac{em}{2\pi^2\hbar^3}(k_B T)^2 \, e^{-\phi / k_B T} .$$

Hint: Use spherical coordinates.

9C. The diagram below shows the band structure of a square lattice in the empty-lattice approximation.
a. Label each of the bands with the corresponding **G**.
b. Draw the reciprocal lattice, and indicate the wave vectors (starting from the origin) corresponding to the four points marked A, B, C, and D.