Physics 322a-2007 Solid State Physics Walter F. Smith Haverford College

Assignment 8

Due: Friday, Nov. 9 at 4 pm

Reading: Livingston Ch. 12

Assigned exercises (group problems unless otherwise noted):

8A. Individual problem Fermi gases in astrophysics (Adapted from Kittel problem 6.4)
a. Estimate the number of electrons in the Sun. In a white dwarf, this number of electrons may be separated from their nuclei and contained in a sphere of radius 2_7 m. Using the free electron model, find the Fermi energy of these electrons in eV.
b. The energy of an electron in the relativistic limit $E >> mc^2$ is related to the momentum by $E \cong pc$. Show that the Fermi energy for a free electron gas in this limit is $\varepsilon_F = \hbar c\left(3\pi^2 N_e\right)^{1/3}$.
c. If the above number of electrons were contained within a pulsar of radius 10 km, show that the Fermi energy would be about 4_8 eV (your answer might be different from this by up to 50%, depending on the assumptions you made in part a). This value explains why pulsars are believed to be composed largely of neutrons rather than of protons and electrons, for the energy release in the reaction $n \rightarrow p + e^-$ is only 8_5 eV, which is not enough to enable many electrons to form a Fermi sea. The neutron decay proceeds only until the electron concentration builds up enough to creat a Fermi level of 8_6 eV, at which point the neutron, proton, and electron concentrations are in equilibrium.

8B. Using the free electron approximation, obtain expressions for the percentage change in Fermi energy caused by

a. thermal expansion ($\frac{\Delta L}{L} = \alpha\,\Delta T$, where α, the thermal expansion coefficient, is assumed to be constant, and is characteristic of the material)

b. hydrostatic pressure ($\frac{\Delta V}{V} = -\kappa\,\Delta P$, where V is the sample volume, P is the pressure and κ, the isothermal compressibility, is assumed to be constant, and is characteristic of the material)

c. For Na (α = 7_{-5} K^{-1}) and for Cu (α = 1.7_{-5} K^{-1}), cacluclate the percentage change in Fermi energy per degree increase in temperature. *(Answers: Na: -0.014% Cu: -0.0034%)*

d. For Na (κ = 14.7_{-11} Pa^{-1}) and for Cu (κ = 7.3_{-12} Pa^{-1}), calculate the hydrostatic pressure change needed to change the Fermi energy by a factor 1.000001.

Livingston:
12-3 Individual problem (Note: Livingston is measuring the Fermi energy from the bottom of the conduction band, as shown in Fig. 12.7, p. 225)

12-9 part a only

12-11

13-10 (This does not actually require any concepts from Ch. 13.)