Physics 322a-2007 Solid State Physics Walter F. Smith Haverford College

Assignment 11 (Corrected)

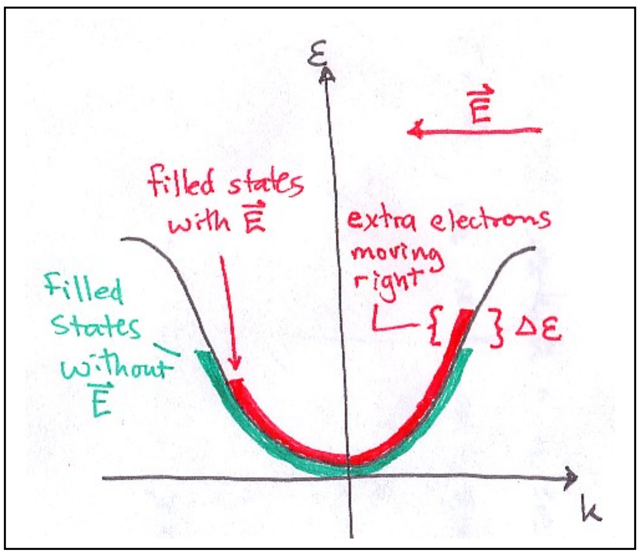
Due: Friday, 12-14-07 at 4 pm.

Reading: Livingston Ch. 16

Assigned exercises (all group problems, except as noted)

Livingston
14-9 (Hint: Think about the nearly free electron model, and think about density of states.)
16-1
16-2 (Hint: Note that one side is much more heavily doped than the other, and use this to make a suitable approximation)
16-5
16-9 (individual problem)

11A. The dispersion relation for the lowest band of a hypothetical one-dimensional metal has the form $E = 2A \sin^2 (ka/2)$ in the repeated zone scheme, where a is the crystal lattice spacing.
a. For low wavevector, the effective mass m^* is found to be the same as the free electron mass. What must be the value of A?
b. Derive an expression for m^* as a function of k for this system, and sketch the relation. Indicate zone boundaries explicitly on your sketch.
c. The term "density of states" sometimes refers to the number of states per energy, and sometimes to the number of states per energy per volume. In this case, we'll use the latter definition. Let the density of states $Z_e(\varepsilon)$ be defined by $Z_e(\varepsilon)\, d\varepsilon$ = (number of states per length) in the energy range ε to $\varepsilon + d\varepsilon$.. Derive an expression for $Z_e(\varepsilon)$, and evaluate it for energies corresponding to $k = \pi / 2a$ and $k = \pi / a$. (Hint: the distribution of states in k-space in the extended zone scheme is determined by the Born-von Karman boundary conditions, and is unaffected by the details of the band structure.)



11B. The diagram to the right shows the dispersion relation for a one-dimensional material, with a band that is part full. The states shaded in green are occupied. Upon application of an external electric field, the occupied states shift, so that now the states shaded in red are occupied; as you can see, there are now more electrons moving right, and fewer moving left, leading to a net current. Use the fact that the work done by an external force on an electron during the elastic collision time (for an electron at the Fermi energy) τ_F is given by Work = Power × time = $(F_{ext} v)\tau_F$, and that this work equals the shift in energy shown in the diagram (for the case of an externally applied electric field) to show that $\sigma = 2 v_F^2 \, Z(\varepsilon_F) e^2 \tau_F$, which is the 1D version of Livingston's equation 14.2 (p. 258). Assume that $\Delta\varepsilon$ is small, so that all the electrons involved in the shift can be assumed to move at v_F . Also, note that the Z that Livingston uses in this equation is density of states divided by the volume of the sample.