

Physics 322a – 2007 Practice Problems for Exam 2

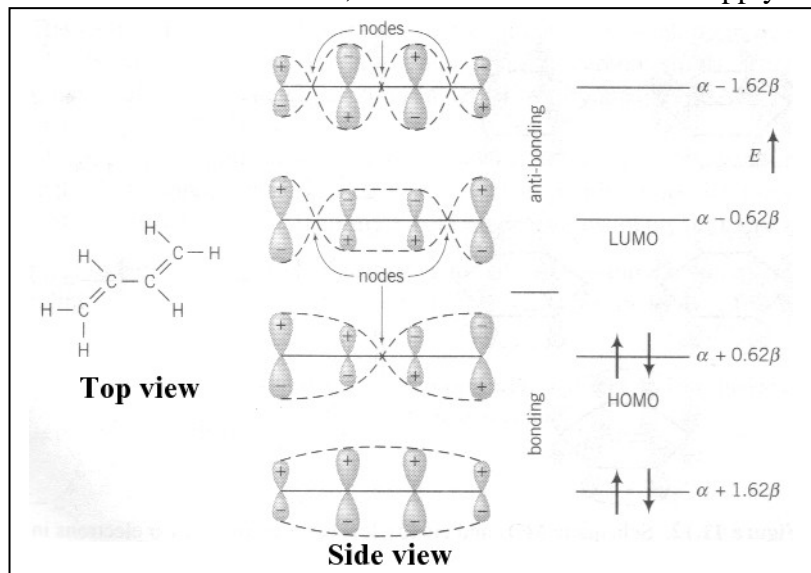
You should be able to do each of these problems (including all sub-parts) in about 15 minutes or less. (One exception: you should be able to do problem 6 in 25 minutes.) If it takes you longer, you have not mastered the associated material thoroughly enough. **Note:** Of course, these problems don't cover exactly the same topics as the questions on the real exam. Therefore, doing well on these practice problems does not insure that you are adequately prepared for the exam. However, doing poorly on these problems does mean that you need to study more. **You will need a ruler marked in mm and cm.**

1. You apply a voltage of 100 V along the length of a conductor of dimensions 10 cm x 1 mm x 1 mm. The relaxation time τ is $1.00 \cdot 10^{-14}$ s. If you apply a magnetic field of 1.00 T perpendicular to one of the side faces, what will be the Hall voltage (i.e. the transverse voltage), assuming this material is well described by the Drude model?

2. Explain what the reduced zone scheme is, and why it is possible to represent all possible electron wavefunctions of a crystal in this scheme. You may assume that your audience fully understands the extended zone scheme, and you may make use of either form of Bloch's theorem in formulating your argument. Your discussion should be clear and thorough.

3. **Energy levels of Butadiene** As you'll recall from class, the tight-binding approximation is based on approximating the crystal orbitals by forming linear combinations of atomic orbitals. When this approach is applied to finite systems, such as a molecule, it is called the "Linear Combination of Atomic Orbitals", or LCAO approach. For the crystal, we used Bloch's theorem to find the coefficients in the linear combination. However, Bloch's theorem does not apply to finite systems such as molecules. Therefore, we have to find the coefficients by other means.

Shown here is a section of a page from your text, showing the butadiene molecule schematically in a top view and in a side view. We will consider only the electrons in the molecular orbitals derived from the $2p_z$ atomic orbitals, which project above and below the plane of the molecule, as shown in the side view. Combining the four atomic



orbitals (from the four carbon atoms) creates four molecular orbitals, with the energies shown. The LCAO approximation for the ground state of the molecule is given by

$$\psi = \frac{1}{\sqrt{2 + 2\gamma^2}} (\varphi_1 + \gamma \varphi_2 + \gamma \varphi_3 + \varphi_4), \text{ where } \gamma = 1.618.$$

In this expression, $\varphi_1 = \varphi(\mathbf{r})$ is the atomic orbital centered on the leftmost nucleus (which we'll take as the origin), $\varphi_2 = \varphi(\mathbf{r} - \mathbf{T}_2)$ is the atomic orbital centered on the next nucleus to the right, $\varphi_3 = \varphi(\mathbf{r} - \mathbf{T}_3)$ is the atomic orbital centered on the next nucleus to the right, and $\varphi_4 = \varphi(\mathbf{r} - \mathbf{T}_4)$ is the atomic orbital centered on the rightmost nucleus. (The atomic orbitals of the middle two atoms are shown larger in the side view to indicate that they are weighted more heavily in the superposition.) Show that the expectation value for the energy of the ground state is as given in the figure, where $\alpha \equiv \int_{\text{all space}} \varphi_1^* H \varphi_1 d^3\mathbf{r}$, $\beta \equiv \int_{\text{all space}} \varphi_1^* H \varphi_2 d^3\mathbf{r}$, the four carbon atoms

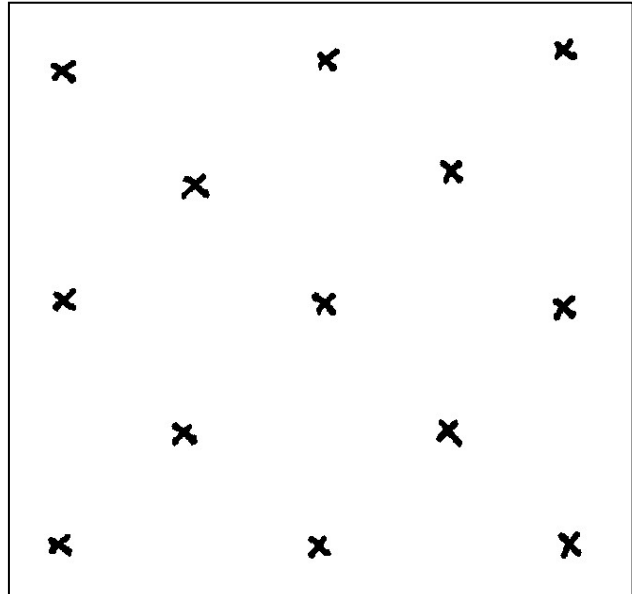
are assumed to be equally spaced, and we are using the Huckel approximation (i.e. ignoring everything beyond nearest neighbor interactions.)

6. A divalent (i.e. two valence electrons per atom) metal has a bcc structure with a lattice constant of $a = 5.23 \text{ \AA}$.

a. Using free electron theory, determine the Fermi energy of this metal; express your answer in eV.

b. The work function for this metal is 2.22 eV. Using the value for E_F from part a, calculate the maximum and minimum wavelengths of photoelectrons emitted from the metal when it is exposed to 7 eV photons.

c. As you should recall by now, the reciprocal lattice for such a system is fcc, with lattice constant $\frac{4\pi}{a}$. Shown here is a slice through this reciprocal lattice. Reproduce this same slice in your exam book, and draw on it the boundaries of the first Brillouin zone (in this plane).



d. Superimpose the free electron Fermi surface on your drawing (i.e. that part of the surface that the plane shown cuts through).

e. For this free-electron picture, describe which bands (if any) are completely filled, and which bands (if any) are partially filled.

f. For the sketch you made in part e, would you predict metallic behavior, or instead semiconducting/insulating behavior? Explain.