

### Physics 322-2007 Practice questions for exam 1

You should be able to do each of these problems (including all sub-parts) in about 15 minutes or less (20 minutes for problem 2). If it takes you longer, you have not mastered the associated material thoroughly enough. **Note:** Of course, these problems don't cover exactly the same topics as the questions on the real exam. Therefore, doing well on these practice problems does not insure that you are adequately prepared for the exam. However, doing poorly on these problems does mean that you need to study more.

**1. (20 points) Formulas for the reciprocal lattice in two dimensions.** A two-dimensional lattice is defined by the primitive vectors  $\mathbf{a}_1 = a_{1x}\hat{\mathbf{x}} + a_{1y}\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$  and  $\mathbf{a}_2 = a_{2x}\hat{\mathbf{x}} + a_{2y}\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$ , which are not necessarily perpendicular or of the same length. (Here, I have followed Kittel's convention of using  $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ , and  $\hat{\mathbf{z}}$  to represent the unit vectors in the  $x, y$ , and  $z$  directions.) Verify that the components of the reciprocal lattice vectors can be found using the following formulas:

$$b_{1x} = 2\pi \frac{a_{2y}}{|\mathbf{a}_1 \times \mathbf{a}_2|} \quad b_{1y} = 2\pi \frac{(-a_{2x})}{|\mathbf{a}_1 \times \mathbf{a}_2|}$$
$$b_{2x} = 2\pi \frac{(-a_{1y})}{|\mathbf{a}_1 \times \mathbf{a}_2|} \quad b_{2y} = 2\pi \frac{a_{1x}}{|\mathbf{a}_1 \times \mathbf{a}_2|}$$

Do not do your verification by starting from the analogous formulas for three-dimensional lattices, but rather by using more fundamental facts about the reciprocal lattice.

**2. (40 points total) Electron diffraction by a surface.** Davisson and Germer, working at Bell Labs in 1926, verified De Broglie's hypothesis that there is a wavelength  $\lambda = \frac{2\pi\hbar}{p}$  associated

with a free electron moving with momentum  $p$ . They did so by observing diffraction of a beam of monoenergetic electrons off a Nickel (111) surface. The low energy, grazing incidence electron beam they used does not penetrate very far at all into the crystal, so the diffraction is dominated by the two-dimensional arrangement of atoms on the surface.

**a. (10 points)** Nickel (Ni) crystallizes in fcc form, with conventional unit cell sidelength 3.52 Å. Sketch the conventional crystal unit cell for this system, and depict the (111) plane on your sketch.

**b. (5 points)** As you should be able to tell from your sketch, the atoms in this plane form a two-dimensional hexagonal lattice. The basis vectors for such a lattice are  $\mathbf{a}_1 = a\hat{\mathbf{x}}$  and

$\mathbf{a}_2 = -\frac{1}{2}a\hat{\mathbf{x}} + \frac{\sqrt{3}}{2}a\hat{\mathbf{y}}$ . Show that  $a = 2.49$  Å.

**c. (8 points)** Using the formulas from problem 1, one finds that the reciprocal lattice vectors for this two-dimensional lattice are given by

$$\mathbf{b}_1 = \frac{2\pi}{a} \left( \hat{\mathbf{x}} + \frac{1}{\sqrt{3}}\hat{\mathbf{y}} \right) \quad \text{and} \quad \mathbf{b}_2 = \frac{2\pi}{a} \frac{2}{\sqrt{3}}\hat{\mathbf{y}}$$

Since the electron beam is at grazing incidence, you may assume that it's essentially parallel to the surface. Show that the minimum energy that an electron must have in order for diffraction to occur from this surface is 8.088 eV. (Recall that  $1 \text{ eV} = 1.60219 \times 10^{-19} \text{ J}$ , and that  $E = \frac{p^2}{2m}$  ).

**d. (7 points)** Assume for the actual experiment that the electrons in the beam have energy 8.247 eV. What is the smallest angle relative to  $\hat{y}$  at which the incoming beam could be in order to get a diffracted beam?

**e. (5 points)** At what angle relative to  $\hat{y}$  would the diffracted beam go?

**f. (5 points)** Sketch the Brillouin construction for this diffraction event. (Your sketch need not be to scale.) Note: you may well have already made such a sketch in order to do some of the preceding parts.

**3. (25 points)** For phonons which can be modeled by the harmonic approximation we used in class, at approximately what ratio of wavelength  $\lambda$  to interplane spacing  $a$  will the group velocity be 5% more than the phase velocity? *Hint: For  $x$  less than about 0.5,  $\tan x \approx x + \frac{x^3}{3}$ .*