Physics 322a – 2007 Exam 2 Coverage

Exam dates: distributed Monday 11-26-07, due at 4 pm on the following Friday (11-30-07). This exam is cumulative, but will focus primarily on the material covered since the last exam:

Chapters covered:
Livingston chapters 1, 12
Note: much of what we covered during this period is not in the text, so you will need to review your lecture notes carefully

Classes covered: Class 13 (Monday 10-1-07) through Class 30 (Friday 11-16-07), through and including empty lattice model.

Assignments covered: 6-9

Equation sheet: You will be allowed 35 equations.

Most important topics (the exam is not necessarily limited to these topics, but these are the highlights of what we have done since the last exam):

Drude model

- Basic assumptions
- Definition of τ
- Be ready to derive the expression for the resistivity from the basic assumptions
- Drift velocity
- Elastic mean free path
  - Definition
  - Why the experimental values of it imply that the electron doesn't collide with nuclei that are part of the regular crystal lattice.
- Be familiar with the relation between voltage and electric field
- Know the definition of current density
- Microscopic version of Ohm's law $E = j\rho$
- Mobility
- Hall Effect

Tight Binding Model

- Understand the basic idea (i.e. that it's LCAO applied to crystals)
- Understand that the result has the form of a "discretized plane wave":

$$\psi_{\mathbf{k}}(\mathbf{r}) = N^{-\frac{1}{2}} \sum_j e^{i\mathbf{k}\cdot\mathbf{T}_j} \varphi(\mathbf{r} - \mathbf{T}_j)$$

- Be able to use the equation for the expectation value of the energy:

$$\langle E \rangle = \int_{\substack{all\\space}} \varphi^*(\mathbf{r}) H \varphi(\mathbf{r})\, d^3\mathbf{r} + \sum_{\substack{\mathbf{T}_{nn}=\text{n.n.}\\ \text{to origin}}} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{T}_{\mathrm{nn}}} \int_{\substack{all\\space}} \varphi^*(\mathbf{r} - \mathbf{T}_{nn}) H \varphi(\mathbf{r})\, d^3\mathbf{r}$$

- Using the very rough approximation $H\varphi(\mathbf{r}) \approx H_{atomic}\,\varphi(\mathbf{r})$ to make rough arguments about the energy
- Why the above equation tells us that the weaker the overlap, the narrower the band
- The band derived from an atomic orbital has $2N$ states
- Qualitative argument for the shape of the second band in 1D
- Understand qualitatively why higher bands have higher bandwidth

Metals and Semiconductors

Why band overlap is needed to create a divalent metal

Bloch Theorem

Understand both versions: $\psi(\mathbf{r}+\mathbf{T}) = e^{i\mathbf{k}\cdot\mathbf{T}}\psi(\mathbf{r})$ and $\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u(\mathbf{r})$

Be able to derive the second version from the first

Meaning:

1) $\mathbf{k}\cdot\mathbf{T}$ is the quantum mechanical phase accumulated as you go from one point in a unit cell to the equivalent point in a different unit cell which is **T** away

2) Bloch states are not momentum eigenstates

3) Changing **k** by a reciprocal lattice vector doesn't change ψ (This is analogous to saying that changing an angle by 360^o doesn't change its physical significance.)

4) The meaningful range of **k** is the first Brillouin zone. (This is analogous to saying that the meaningful range of an angle is 0 to 360^o.)

Understand the tradeoff in "wiggliness" between the $e^{i\mathbf{k}\cdot\mathbf{r}}$ part of $\psi(\mathbf{r})$ and the $u(\mathbf{r})$ part when **k** is changed by a reciprocal lattice vector

Brillouin zones

How the first Brillouin zone is defined

How higher order Brillouin zones are defined

Each BZ has the same volume

Each BZ can "hold" 2 electrons/unit cell

Any **k** outside the 1BZ is physically equivalent to one inside the 1BZ: $\mathbf{k} + \mathbf{G} = \mathbf{k}_{1BZ}$

Extended, Reduced, and Repeated Zone representations

Free electron model

Born-von Karmann (i.e. periodic) boundary conditions

"volume" per allowed **k**

Density of states in 3D

Understand in detail why higher bands have higher bandwidth

Empty lattice model

What it is, how to use it to roughly predict band structures

Why, in 1D, it predicts a band structure which is qualitatively similar to that from tight binding

Each band can "hold" 2 electrons/unit cell

Each band occupies a "volume" equal to the 1BZ => One can make a one-to-one association between bands and BZs (i.e. second band corresponds to second BZ, etc.)

Understand how multiple bands are represented in the reduced zone scheme (i.e. that each **k** in the 1BZ can be associated either with a state in band 1, or with a state in band 2, etc.)

Extended, reduced, or repeated zone schemes

Fermi level topics (covered in reading and assignments only)

Contact potential

Thermionic emission, including Richardson-Dushmann equation

Photoelectron spectroscopy

Fermi surface

Definition

Know what the free-electron Fermi surface looks like in 1D, 2D, and 3D, be able to calculate its radius, given the number of electrons per unit cell

Volume inside Fermi surface is conserved when interactions are turned on

Band diagrams

How to construct them in the empty lattice model

Understand what the horizontal scale signifies

Understand why there are multiple curves

Be able to use a band diagram with the Fermi energy marked to determine where there would be a peak in the optical absorption

Understand what an "absorption edge" is. Be ready to answer questions such as, "Why can't the blue coloration of an object be due to an absorption edge?"

Be able to use a band diagram with the Fermi energy marked to determine whether a substance is metallic

Direct (no change in k-needed for lowest energy transitions) and indirect bandgaps

Heat capacity of electrons

Be able to use equation 12.26

Fermi-Dirac distribution

$$F = \frac{1}{e^{(\varepsilon - \varepsilon_F)/k_B T} + 1}$$

Know how variation of T affects the graph of this.