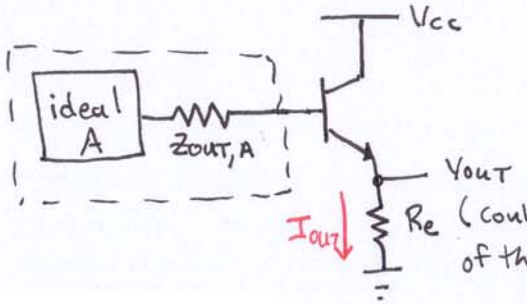
An example of calculating output impedance for circuits including active components (such as transistors): output impedance of an emitter follower



Strategy: "Apply" ΔV_{OUT}, find resulting ΔI_{OUT}, use $Z_{OUT} \equiv -\Delta V_{OUT}/\Delta I_{OUT}$

$V_{be} = 0.6V \Rightarrow \Delta V_b = \Delta V_{OUT} \Rightarrow \Delta I_b = -\Delta V_{OUT}/Z_{OUT,A}$

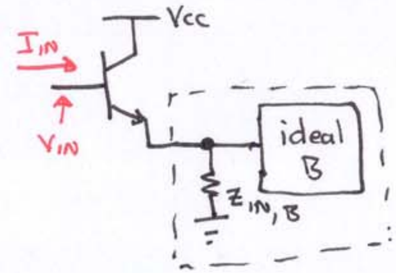
$I_c = \beta I_b \Rightarrow \Delta I_c = \beta \Delta I_b = -\beta \Delta V_{OUT}/Z_{OUT,A}$

$\Delta I_{OUT} = \Delta I_e = \Delta I_b + \Delta I_c = -(1+\beta)\Delta V_{OUT}/Z_{OUT,A}$

$$\Rightarrow Z_{OUT} = -\frac{\Delta V_{OUT}}{\Delta I_{OUT}} = \frac{Z_{OUT,A}}{1+\beta}$$

$\Rightarrow$ output impedance reduced by $(1+\beta) \approx 100$.

An example of calculating input impedance for circuits including active components: input impedance of an emitter follower



Strategy: apply ΔV_{IN}, find resulting ΔI_{IN}, use $Z_{IN} \equiv \Delta V_{IN}/\Delta I_{IN}$

$V_{be} = 0.6V \Rightarrow \Delta V_{OUT} = \Delta V_{IN}$

$\Rightarrow \Delta I_{OUT} = \Delta V_{IN}/Z_{IN,B}$

But we also have $\Delta I_{OUT} = \Delta I_b + \Delta I_c$, $\Delta I_c = \Delta I_b \cdot \beta$ $\Rightarrow \Delta I_{OUT} = \Delta I_b(1+\beta)$

$$\Leftrightarrow \Delta I_b = \frac{\Delta I_{OUT}}{1+\beta} = \frac{\Delta V_{IN}}{Z_{IN,B}(1+\beta)}$$

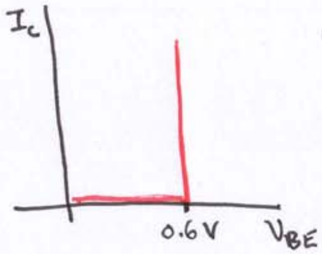
$\Delta I_{IN} = \Delta I_b$

$$Z_{IN} = \frac{\Delta V_{IN}}{\Delta I_{IN}} = Z_{IN,B}(1+\beta)$$

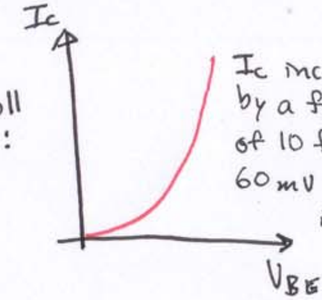
$\Rightarrow$ input impedance increased by $(1+\beta)$!

Ebers-Moll model for transistors

Our simple model:



Reality, Ebers-Moll model:



I_c increases by a factor of 10 for each 60 mV increase in V_{BE}

"transconductance" $g_m \equiv \frac{dI_c}{dV_{BE}}$ is large for bipolar transistors

$$I_c = I_s \left(e^{V_{BE}/V_T} - 1\right)$$

↑ depends strongly on T, transistor type

$V_T \equiv \frac{kT}{e} = 25.3\ \text{mV}$ at room temp. ⇒ typically, $e^{V_{BE}/V_T} \gg 1$, $I_c \cong I_s e^{V_{BE}/V_T}$

⇒ $V_{BE} \cong V_T \ln \frac{I_c}{I_s}$