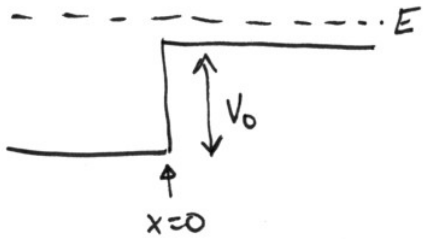
# PHYSICS 302b-2010 CLASS 20 THURSDAY 4-8-10 SUMMARY

## Scattering & Tunneling (ctd.)




$E > V_0$

→ To left of step $\psi = Ae^{ikx} + Be^{-ikx}$ $\quad k \equiv \sqrt{2mE}/\hbar$

To right of step, $\psi = Ce^{iQx}$ $\quad Q \equiv \sqrt{2m(E-V_0)}/\hbar$

B & C can be determined in terms of A using continuity of $\psi$ & $d\psi/dx$ at $x=0$.

→ For a right-propagating wave $\psi = Ae^{ikx}$, $j = |A|^2$.

For a left-propagating wave $\psi = Be^{-ikx}$, $j = -|B|^2$

For $E < V_0$ (classically, particle is forbidden from $x>0$)

→ to left of step: Same as above

to right of step, $\psi = \underbrace{Ce^{-Kx}}_{\text{"evanescent wave"}} + \cancel{D}e^{Kx}$ where $K \equiv \sqrt{2m(V_0-E)}/\hbar$

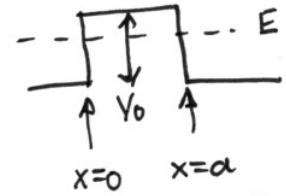
$D \to 0$ to avoid divergence of $\psi$ in the classically forbidden region

"evanescent wave" — not a propagating wave

→ $j=0$ for $x>0$, as expected.

Used continuity of $\psi$ & $d\psi/dx$ at $x=0$ to find B and C in terms of A, using Mathematica.

## Tunneling




$x<0$: $\psi = Ae^{ikx} + Be^{-ikx}$

$0<x<a$: $\psi = Ce^{-Kx} + \underbrace{De^{Kx}}_{}$

can't discard, since this solution is only valid for $x<a$

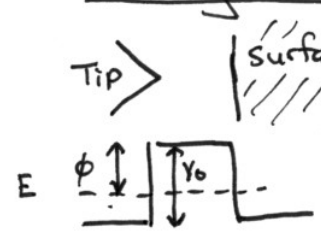
$x>0$: $\psi = Fe^{ikx}$

Apply continuity of $\psi$ & $d\psi/dx$ at $x=0$ & $x=a$ to find B, C, D, & F in terms of A (using Mathematica)

→ For rare tunneling (i.e. for $Ka \gg 1$), the probability of transmission is

$$\boxed{T \equiv \frac{j_{trans}}{j_{incident}} \cong \left(\frac{4kK}{k^2+K^2}\right)^2 e^{-2Ka}}$$

## Scanning Tunneling Microscopy (STM)




Workfunction $\phi \equiv$ the amount of energy needed to extract the most energetic electron from the tip to the vacuum

→ $\dfrac{\text{tunneling prob for closest atom}}{\text{" " for next " "}} \simeq 400$