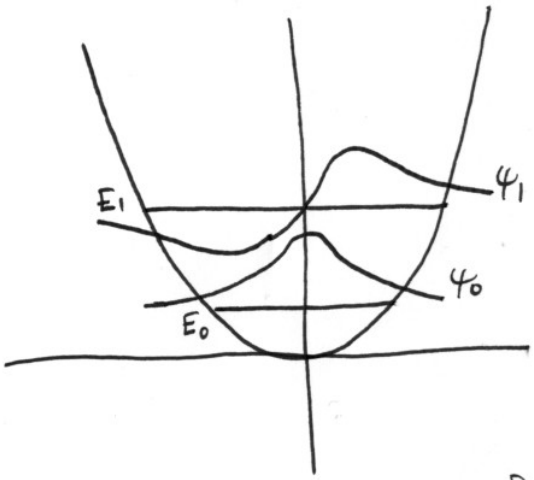
PHYSICS 3026-2010 CLASS 22 THURSDAY 4-15-10 SUMMARY

Harmonic oscillator wavefunctions

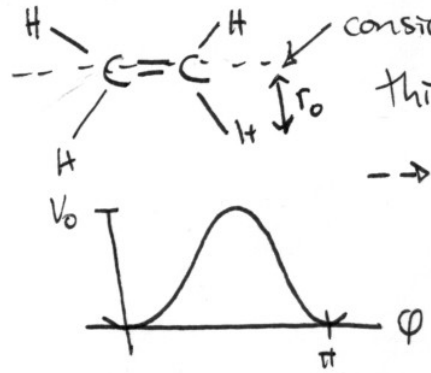
$\hat{a}|0\rangle = 0 \rightarrow \psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$

To find the others, we use the raising operator, e.g. $\hat{a}^+|0\rangle = \sqrt{0+1}\,|1\rangle$

$\rightarrow \psi_1 = \left(\frac{m\omega}{\hbar}\right)^{3/4} \frac{\sqrt{2}}{\pi^{1/4}} x e^{-m\omega x^2/2\hbar}$



Example: Torsional motion of ethylene



consider twist φ about this axis

$\rightarrow \hat{H} = 4 \cdot \underbrace{\frac{\hat{L}^2}{2I}}_{\text{KE of one hydrogen}} + V_0\varphi^2$

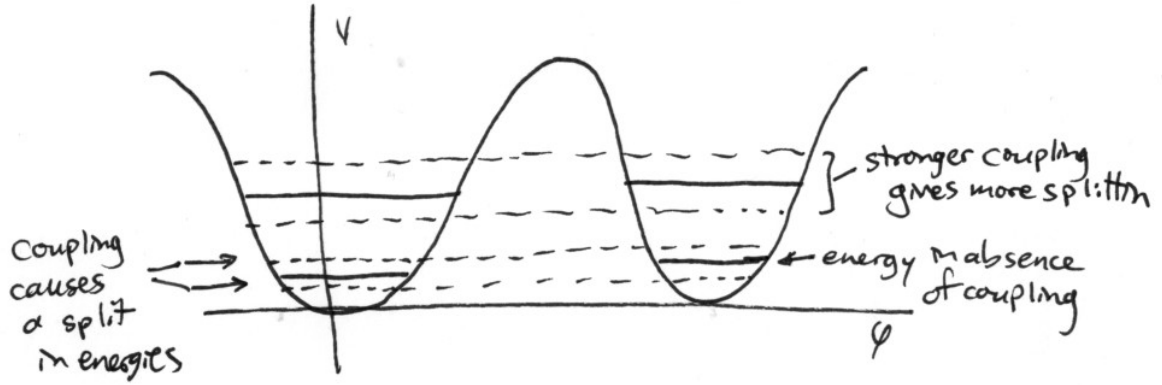
p is generator of translations $\leftrightarrow$ L is generator of rotations

$[\hat{x}, \hat{p}] = i\hbar \leftrightarrow [\hat{\varphi}, \hat{L}] = i\hbar$

$E = (n+\frac{1}{2})\hbar\omega \leftrightarrow E = (n+\frac{1}{2})\hbar\omega$

$\omega = \sqrt{\frac{V_0}{2m_H r_0^2}}$

Coupling between wells gives splitting, as for NH_3!



Position & Momentum in 3D

$\hat{I} = \int d\vec{r}\,|\vec{r}\rangle\langle\vec{r}|$ $\qquad [\hat{p}_x, \hat{p}_y] = 0$ (You'll show)

$\rightarrow \hat{T}(\vec{a}) = e^{-i\hat{\vec{p}}\cdot\vec{a}/\hbar}$ where $\hat{\vec{p}} \equiv \hat{i}\hat{p}_x + \hat{j}p_y + \hat{k}p_z$