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Energy eigenstates

$$\hat{H}|E\rangle = E|E\rangle$$

set $|\psi(t=0)\rangle = |E\rangle$

$$\rightarrow {}^{*}\boxed{|\psi(t)\rangle = e^{-iEt/\hbar}|E\rangle}^{*}$$

The time progression of an energy eigenstate is only the overall phase factor $e^{-iEt/\hbar}$

However, by superposing energy eigenstates, can get interesting time behavior

e.g. $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|E_1\rangle + |E_2\rangle)$

$$\rightarrow |\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-iE_1t/\hbar}\left(|E_1\rangle + \underbrace{e^{-i\Delta E t/\hbar}}_{\text{important relative phase factor}}|E_2\rangle\right)$$

$\Delta E \equiv E_2 - E_1$

Time dependence of expectation values

$$\rightarrow \boxed{\frac{d\langle A\rangle}{dt} = \left\langle \frac{\partial \hat{A}}{\partial t}\right\rangle + \frac{i}{\hbar}\langle[\hat{H},\hat{A}]\rangle}$$

If $\hat{A}$ has no explicit time dependence, $\frac{\partial \hat{A}}{\partial t} = 0$, so if $[\hat{H},\hat{A}] = 0$, then $\langle A\rangle = \text{const.}$

Precession

Apply $\vec{B} = B_0\hat{k}$

$\rightarrow$ total energy operator $\hat{H} = \omega_0 \hat{S}_z$

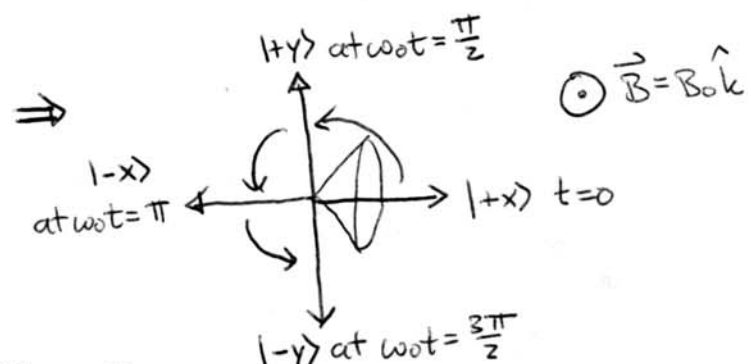
where $\omega_0 \equiv -\frac{gqB_0}{2mc}$

So, $\hat{H}|\pm z\rangle = \pm\frac{\hbar\omega_0}{2}|\pm z\rangle$

Let $|\psi(t=0)\rangle = |+x\rangle$

$$\rightarrow |\psi(t)\rangle = \frac{e^{-i\omega_0 t/2}}{\sqrt{2}}\begin{pmatrix}1\\ e^{i\omega_0 t}\end{pmatrix}$$

$\Rightarrow$



Berry Phase

Note: when $\omega_0 t = 2\pi$, get $-|+x\rangle$. To recover the original state, must rotate by 4π!

Magnetic resonance

For spin $\frac{1}{2}$, $\vec{B} = B_0 \hat{k}$, $E = \pm \frac{\hbar}{2}\omega_0$

where $\omega_0 = \dfrac{-q g B_0}{2mc}$ $\quad g = \begin{cases} 2.002\ldots \text{ for electron} \\ 5.586\ldots \text{ for proton} \end{cases}$

Now apply linearly polarized light at ω:

$\vec{B} = B_0 \hat{k} + B_1 \cos\omega t \, \hat{i}$

$\Rightarrow \hat{H} = \omega_0 \hat{S}_z + \omega_1 \cos\omega t \, \hat{S}_x$ where $\omega_1 \equiv \dfrac{-g q B_1}{2mc}$

SEQ $\rightarrow$ $i\hbar \begin{pmatrix} \dot{a} \\ \dot{b} \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} \omega_0 & \omega_1 \cos\omega t \\ \omega_1 \cos\omega t & -\omega_0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$

where $a(t)$ & $b(t)$ are to be determined