

# 1. SOUND AND THE SCIENCE OF MUSICAL INSTRUMENTS

## *Introduction*

In this laboratory you will learn how your own direct experiences relate to the science of music. You will perform simple measurements to become acquainted with the concepts of frequency, wavelength, and waveforms for harmonic oscillators (tuning forks) and more complex sources (the human voice and musical instruments.) We will see how the resonant frequencies of string and wind instruments determine their musical character, and examine the mathematical foundation of the harmonic structure of Western music. Throughout, the mathematical operation of Fourier transforming the waveforms of sound will allow you to interpret the harmonic structure of even complex sounds.

You must read through section 2, *Background*, and work out the pre-lab problems in advance. Bring in your favorite musical instrument and come prepared to sing (and listen). If you don't play an instrument, we'll provide several instruments.

**Associated Readings in Wolfson & Pasachoff:** Chapters 16 & 17, especially sections 16-2, 16-6, & 17-6.

**Associated Readings in Hecht:** Chapter 11, especially sections 11-9 and 11-10

**No quantitative uncertainty analysis is expected for this lab.**

## *Background*

### **Equipment Operation**

The microphone you will use contains a thin membrane that can vibrate in response to the incident sound waves. This vibration produces a time-varying electrical signal. By sensing this variation with an electronic circuit, we can convert the fluctuations in air pressure, which make up the sound wave, into time variations in an electric voltage or current. What is actually being measured and displayed on your computer screen is this voltage output of the microphone. When plotted versus time, it gives a picture of the time variation of the sound wave's amplitude.

You use the detector in combination with the Logger Pro software on your computer. Before attempting the experiments below, you should examine the features described below when you first arrive in lab.

**Pre-lab question 1:** How can you measure the frequency of the sound produced by a tuning fork? Use the sample waveform in Figure 1. How would you compute the wavelength of the tuning fork's sound in air? (The speed of sound in air is 344 m/s.) Measure both the frequency and wavelength in air for Figure 1.

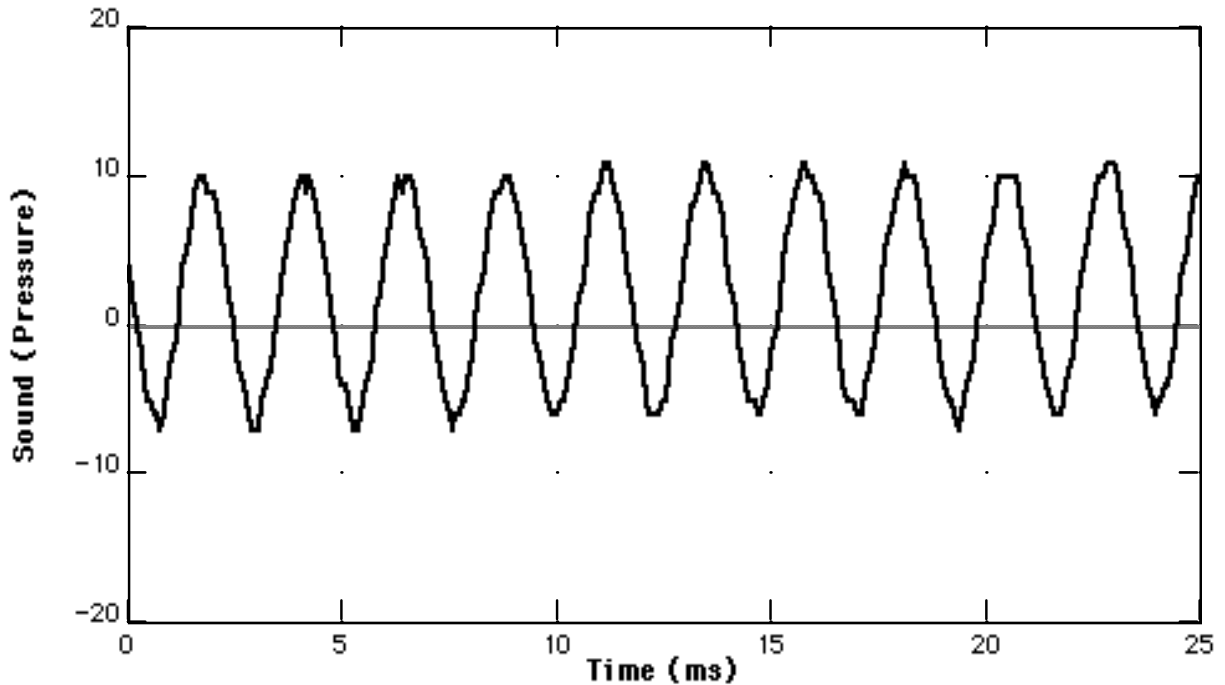


Figure 1: Sample waveform for pre-lab question 1.

### Fourier Series (Amplitude Spectrum)

A musical note in a given octave (say, middle C) has the same period and fundamental frequency regardless of the voice or instrument that produces it. In fact, we define musical notes by their frequency, which approximates what we sense as pitch. The tuning fork produces close to a pure harmonic wave made up of a single sine wave. However, your voice or a musical instrument does not produce a single sine wave only. Each of these cases also has faster time behavior in addition to the periodic structure at its fundamental frequency. Surprisingly, we can still describe these complex waveforms using the language of harmonic waves. A complex waveform can be described as a sum of many harmonic waves with different amplitudes and frequencies; this is shown in Figure 2.

Figure 2a is the sum of the harmonic functions plotted in Figures 2b through 2e. The equation plotted in figure 2a is:

$$y(t) = 2\sin(2\pi * 110s^{-1}t) + 3\sin(2\pi * 220s^{-1}t) + \sin(2\pi * 440s^{-1}t) + \frac{1}{2}\sin\left(2\pi * 880s^{-1}t + \frac{\pi}{4}\right)$$

(1)

The description of a complicated signal in terms of a sum of harmonic waves is referred to as a Fourier series. Fourier series have wide applications in physics, chemistry, and medicine, including their use in image analysis, spectroscopy, and magnetic resonance imaging (MRI). In this lab, we'll look only at one

feature of the Fourier series (Figure 3). This is a plot of the *amplitudes vs. frequencies* of the frequencies that constitute the original waveform. For example, in the case above, we have the following amplitude spectrum: since the note is an A, with fundamental frequency at 110 Hz, there is a large peak at approximately 110 Hz. In addition, to get the exact shape of the waveform, we must also add in peaks at twice, four times, *etc.* the fundamental. These additional peaks give the sound its distinctive quality.

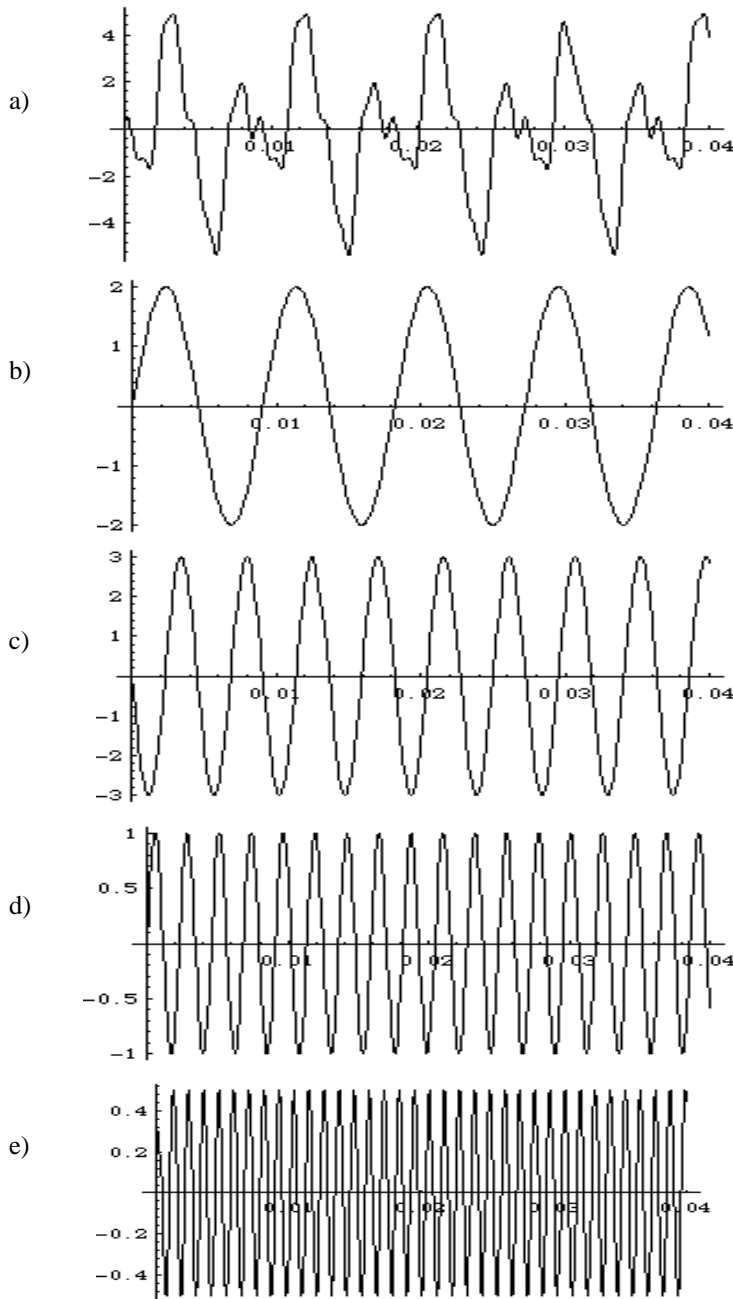


Figure 2: Plot of pressure versus time (in seconds). The waveform in (a) is the sum of the harmonic functions plotted in (b) through (e). Note the differences in the pressure scales for the graphs, as well as the difference in their phases.

Your Logger Pro software performs this computation using a program referred to as FFT (for fast Fourier transform). The FFT is simply a particularly efficient algorithm for computing the amplitude spectrum. We will use the term FFT as shorthand for the spectrum of amplitude vs. frequency.

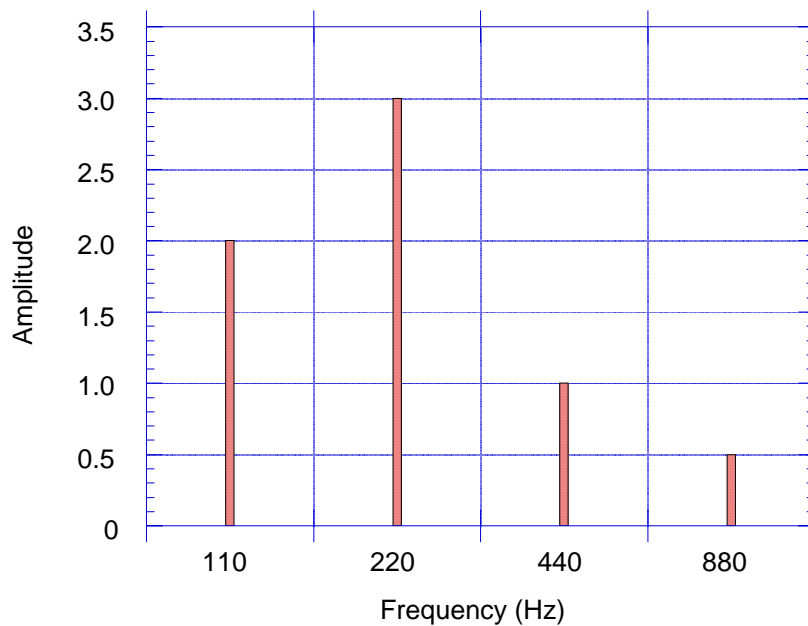
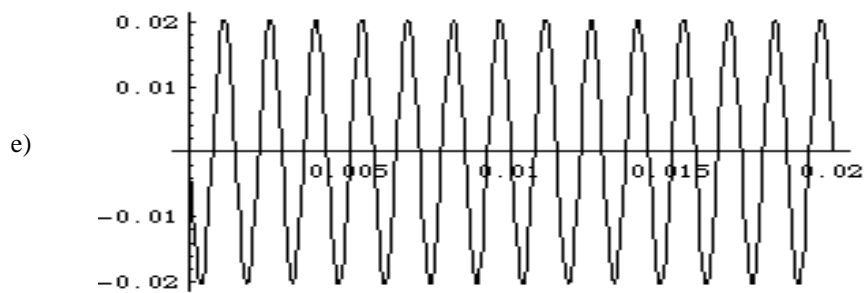
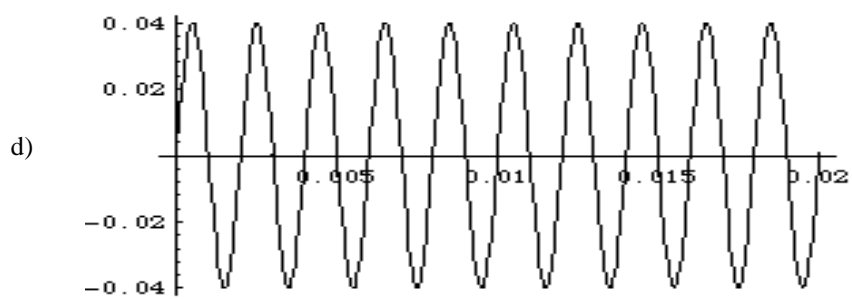
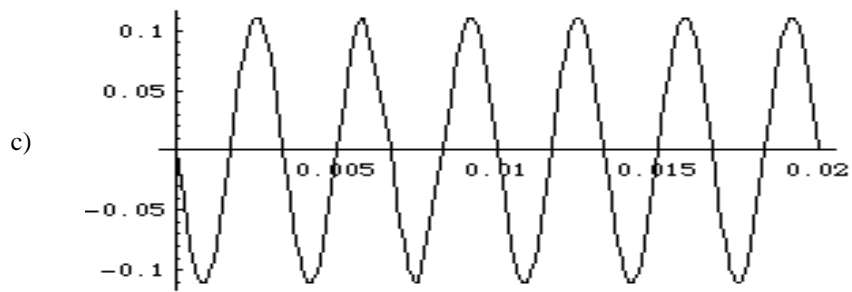
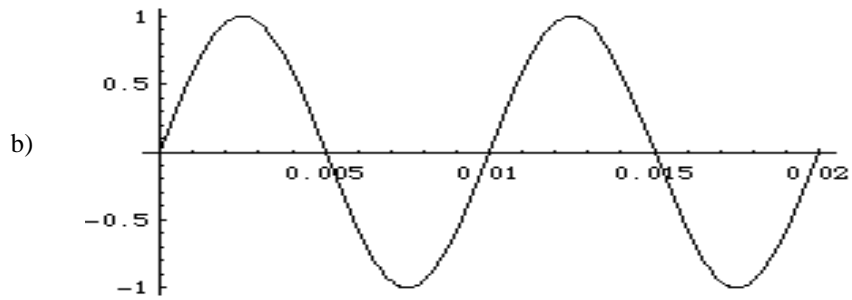
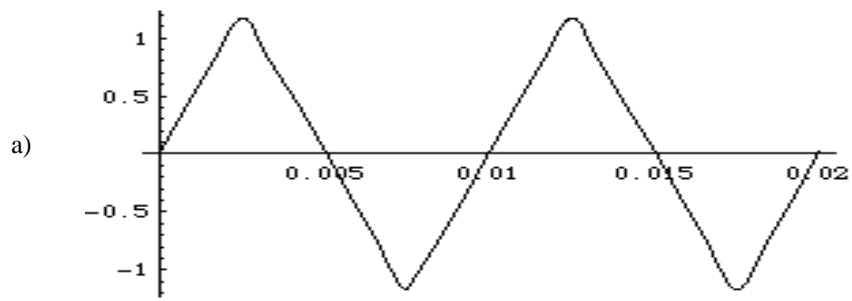


Figure 3: Amplitude vs. frequency spectrum (Fourier coefficients) of waveform in Figure 2a

**Pre-lab question 2:** Deduce and plot the FFT (amplitude vs. frequency) of the waveform in Figure 4a. Its breakdown into sine waves is shown in Figures 4b through 4e.



Time (seconds)

Figure 4: Waveform (pressure vs. time) and its breakdown for pre-lab question 2.

## Beats

Musicians sometimes use the phenomenon of beats to tune instruments. This phenomenon also excellently demonstrates the principle of superposition. When two harmonic waves of the form:

$$p_1 = p_o \sin(2\pi f_1 t) \quad (2a)$$

$$p_2 = p_o \sin(2\pi f_2 t) \quad (2b)$$

are added together, the result can be expressed as

$$p = p_1 + p_2 = 2p_o \cos\left(2\pi \frac{1}{2} \Delta f t\right) \sin(2\pi \bar{f} t), \quad (3)$$

where  $\Delta f = f_1 - f_2$  and  $\bar{f} = (f_1 + f_2)/2$ . You would perceive the summed waveform from equation (3) if two instruments played the two notes simultaneously. Its characteristic behavior is shown in Wolfson & Pasachoff's Figure 16-23 and Hecht's figure 11-41. In addition to a fast oscillation at the mean frequency ( $\bar{f}$ ), there is a gradual modulation of the amplitude of the sound, referred to as *beating*, with a frequency equal to the difference of the two frequencies ( $\Delta f$ ). When the two frequencies being played are very far apart,  $\Delta f = (f_1 - f_2)$  is a large number, and the high frequency beating is not noticeable. However, for closely matched, but not identical frequencies  $f_1$  and  $f_2$ , differences on the order of one hertz are possible. Beating due to these differences is quite audible and rather annoying.

## Experimental Procedure

### Experiment 1: The Nature of Sound Waves

A few features of the software you need to understand straight off are:

- Plug the microphone into CH1 port of the LabPro.
- You can collect a sound signal (a "waveform") right away. When you go to start Logger Pro, instead of clicking on the Logger Pro icon, look for a file on the desktop named SoundLab. If you click on this, it will bring up the graphs you will need to use (already set to an appropriate scale), and you will see a sample graph of someone playing a recorder (Don't worry about recording over this). When you're ready to start recording, click on the Collect button at the upper right-hand side of the screen. You'll need to make noises into the microphone for the computer to collect data. Only reasonably loud noises will trigger the computer to record and plot data. Try this feature out now.

- With a waveform on the screen, try out the Examine feature under the Analyze menu. This allows you to scan a cursor over the screen and make accurate measurements of voltage and time.
- You can change the scale of the horizontal and vertical axes by clicking on the numbers at the endpoints of axes. This will highlight them, then you can type in the new endpoint and hit return.
- The FFT graph at the bottom displays the *amplitude vs. frequency* spectrum of your data automatically.
- To print out a waveform, you select Print Screen under the File menu. Ensure that include your names and a title on each plot that you print.

Most of the other options can best be learned by just trying them out or asking your instructor.

Now that you know how to use the program, let's do some analysis with a tuning fork. Using the tuning fork set, explore the waveforms generated when you *gently* hit a tuning fork, then observe its sound output by holding it near the microphone.

Find the frequency and wavelength (in air) for three different tuning forks. Use your approach from the prelab problem first, using the original waveform to show how you can measure frequency from this. Then, use the FFT's to get the frequency. How well do they agree? What does the FFT for a tuning fork look like? How do these measured frequencies correspond to the rated frequencies?

How does the tuning fork deform to generate sound? You can observe the deformations of the fork by looking at it under a strobe light in the interior room of Harris 105. Why does the tuning fork sound different when it is first struck gently on wood versus after it has been vibrating for a while? (Use the software to capture waveforms just after striking the fork and after it has been vibrating for some time.)

Comment on why you do (or do not!) get differences between the two cases.

For the rest of the lab, you will use your voice and musical instruments. We have these musical instruments available for you to use, but you may bring in your own favorites and substitute them: guitar, ukulele, lap harp, recorder, lollipop drum, claves, flutophone, harmonica, and cowbell. Look to Appendix E for instructions on how to use some of these instruments. And a note (no pun intended!) about the instruments. For the string instruments, you'll get a nicer waveform if you use picks, but if you don't have any, strum the strings with your fingers. Whichever you do, make sure to move the string side to side, not up and down. Also, you might need to hold the instrument right up to the microphone to get a good recording.

## **Experiment 2: Exploring the Character of Notes on Musical Instruments**

Note that our perception of pitch depends upon frequency: we say that a particular frequency tuning fork corresponds to a particular musical note. Now sing a note from the musical scale into your microphone. What does the waveform of your voice look like? Record the waveform of a tuning fork for one musical note (say, middle A) on one graph, then record and display a second waveform of your voice playing the same note on another graph. What differences and similarities do you see between the two graphs? Can you define a way to measure the frequency for these more complex waveforms, similar to that used with the tuning forks above? Do so to measure your frequencies. Now repeat these steps for a musical instrument of your choice, and compare it to both your voice and a tuning fork at the same pitch (if possible). How do the waveforms of different sources playing the same note or pitch compare? What is it about frequency that determines what we call the pitch?

**Optional:** Can you produce the same waveform as the tuning fork by singing? How does your voice's wave form compare to the voice of your lab partner when you are both singing the same note?

## **Experiment 3: Exploring the Frequency Spectrum of Musical Instruments**

Now let's focus on how we can use the FFT, not just to measure frequency, but to understand the complex nature of musical (and other) sounds. Using the Logger Pro software, generate the [FFT](#) of at least four waveforms: the tuning fork, a wind instrument, a string instrument, and a percussion instrument, all producing the same musical note (if possible). Note what differences and similarities occur in the FFTs. Try to find a relationship (that is, a ratio) between the frequencies at which the major peaks in the spectrum of each sound source occur.

Using the theory of string and wind instruments presented in your textbook, explain why they have these relationships. For the string instruments, refer to Hecht, page 480, Fig. 11.45 or Wolfson and Pasachoff Fig. 17-12 on page 427 and the accompanying discussion. Predict what will happen to the fundamental frequency of a string when you hold it firmly halfway down and pluck it, then measure the fundamental of the full string and halved string. Similarly, noting Figs. 11.48-11.50 on pages 483-485 in Hecht or Fig. 17-16 ,page 429 of Wolfson and Pasachoff, explore how the length of a wind instrument determines its allowed frequencies. To do so, predict what will happen when you play your wind instrument with no holes covered, then covering the first three holes nearest the mouthpiece. Make any measurements on the wind instrument you need to help explain your results!

Use your observations above to discuss the differences between sound produced by harmonic instruments (such as the string and wind instruments) and a percussive instrument (like a drum). Relate your thinking to both how they produce sound and what you observe in your measurements.

What do you predict the FFT of a hiss will look like? Of a snap, click or other impulsive sound? Of your voice speaking normally in conversation? *What is the fundamental difference between sounds we perceive as melodious and those we sense as unmusical?*

| Note | Frequency* |
|------|------------|
| C    | 261.7 Hz   |
| C#   | 277.2      |
| D    | 293.7      |
| D#   | 311.2      |
| E    | 329.7      |
| F    | 349.2      |
| F#   | 370.0      |
| G    | 392.0      |
| G#   | 415.3      |
| A    | 440.0      |
| A#   | 466.2      |
| B    | 493.9      |

Table 1: The Chromatic Music Scale. Based on 440 Hz for A (A 440). From *Physics*, Ohanian, W.W. Norton & Co., New York, 1989, p. 441.

**Optional:** Compare the FFT of your voice singing, humming and whistling the same note. Compare the FFT of a string instrument when plucked versus being bowed or strummed. Note that how an instrument is played has a strong influence on the harmonic content (as seen in the FFT) of its waveform. Try blowing very hard on a wind instrument. Is it ever possible to excite non-harmonic frequencies?

**Optional:** Now might be a good time to take a closer look at the string instruments. What part of the instrument produces the sounds that you hear? Take a guess. After you have made your guess, get a tuning fork. Gently strike it against a table, and then hold the bottom of it to the body of a guitar or ukulele or other string instrument. Compare the sound produced by the tuning fork when it is vibrating in air vs. when it is touching the body of the string instrument. What is the wood body doing in the second case? What function do you think the body serves in the string instrument? How does this relate to the answer to the first question?

#### **Experiment 4: Beats**

Use the same musical note from the two different sets of tuning forks. (Choose these so one set is tuned to “concert pitch” while the other uses “physical pitch”. These two systems of tuning give slightly different values for the same note.) If you’re having trouble finding a concert pitch and a physical pitch, just look for two tuning forks with the same note but slightly different frequencies. Try striking both these forks and holding both near your ear, then near the microphone. Notice the warbling noise superimposed on the tone

produced. What frequency values are your two tuning forks rated at? What should be the frequency of the beating? Equation (3) might be helpful. Capture a good waveform of the beating, and measure the frequency of the beating as shown on your plot. Does its value agree with what you expect from  $\Delta f$ ?

**Optional:** You can try generating beats in class using different strings on the same string instrument or by using two different wind instruments. One can hear beats between different instruments not tuned to the same relative pitch; see if you can do this with two different wind instruments playing the same note. One also can hear beating when a guitarist plays 2 octaves higher than the fundamental on the E string and octave plus a fifth on the A string simultaneously. You can do this by lightly holding the D string one-quarter of the way from the top (this establishes a node at that point, forcing the wavelength excited to be one-quarter the size of the fundamental) and lightly holding the E string one-third of the way from the stop (for similar reasons.) These two notes would have identical frequencies if the guitar were in perfect tune, so that  $\Delta f = 0$  and no beating would be heard. If instead the guitar needed tuning, you would hear a distinct beating. A guitarist can adjust the tension in the string to equalize the two frequencies.

### ***Additional Readings:***

#### ***On the Physiology of Sound***

*General Physics*, Morton Sternheim and Joseph Kane, John Wiley, New York, 1991, pp. 564–567.  
(Available in the physics lounge, H107.)

#### **On the mathematics of music**

Eugene Helm, “The vibrating string of the Pythagoreans”, *Scientific American*, December 1967, pg.2.  
F.A. Saunders, “Physics and Music”, *Scientific American*, July 1948, pg. 33.

#### **On particular musical instruments:**

Arthur H. Benade, “The Physics of Brasses”, *Scientific American*, July 1973, pg. 24.  
Arthur H. Benade, “The Physics of Woodwinds”, *Scientific American*, October 1960, pg. 144.  
E. Donnell Blackham, “The Physics of the Piano”, *Scientific American*, December 1965, pg. 88.  
Carleen Maley Hutchings, “The Physics of Violins”, *Scientific American*, November 1962, pg. 79.  
Thomas D. Rossing, “The Physics of Kettledrums”, *Scientific American*, November 1982, pg. 172.  
John C. Schelleng, “The Physics of the Bowed String”, *Scientific American*, January 1974, pg. 87.  
Johan Sundberg, “The Acoustics of the Singing Voice”, *Scientific American*, March 1977, pg. 82.