1. A comet is travelling at 0.6 *c* straight toward the sun. Ten light seconds behind it (as measured in the sun frame), there is a spaceship with scientists aboard, studying the comet; they are travelling with the same velocity as the comet. When the clock on the ship reads "0" (indicating the start of a new day), the ship passes a space buoy that is at rest relative to the sun, and the clock on the buoy reads "0". Shortly after that the scientists see the comet suddenly break in half; they record the time as 20 s. At what time did the comet break in half…

a) …as measured in the buoy frame?

b) …as measured in the spaceship frame?

Solution for part a:

Strategy: i) Find the time when the ship receives the image of the ship breaking up.

ii) In S, the light from the comet approaches the ship at a relative speed of 1.6 *c*. So, we need to subtract $\frac{10c\text{s}}{1.6c} = 6.25$ s to account for the time between when the comet broke up and when the light from this event reached the ship.

Implementing the strategy:

i) We will use the Lorentz transform method to find the time when the ship receives the image of the ship breaking up. Let S be the buoy frame and S' be the ship frame. The event of the ship receiving the image occurs at the ship, which is the definition of $x' = 0$. So,

$x = ?$ $x' = 0$

$t = ?$ $t' = 20$ s

$V = 0.6\,c$

Our desired unknown is *t*, so out of the four Lorentz transform and inverse Lorentz transform equations, the best one is $t = \gamma\left(t' + x'V/c^2\right) = \frac{5}{4}\left(20\text{ s}\right) = 25\text{ s}$.

ii) 25 s – 6.25 s = 18.75 s. This is the time the comet breaks up as seen in the buoy frame.

Solution for part b:

Strategy: i) Find the distance from the ship to the comet as measured in the ship frame.

ii) Use this to compensate for the time it takes the image of the breakup to propagate from the comet to the ship.

Implementing the strategy:

i) The rest length between the comet and the ship is measured in S'. So, $L_{\text{other}} = \frac{L_{\text{r}}}{\gamma} \Leftrightarrow L_{\text{r}} = \gamma L_{\text{other}} = \frac{5}{4}\left(10c\text{s}\right) = 12.5c\text{s}$.

ii) So, it takes 12.5 s for the image to propagate from the comet to the ship, so the time that the comet broke up (as seen in S') is 20 s – 12.5 s = 7.5 s.

Alternate method for part b:

Strategy: i) Find the position of the comet in S when it breaks up, using the known time of this breakup from part a.

ii) Use the inverse Lorentz transform to find the time of the breakup in S'.

Implementing the strategy:

i) The ship passes the buoy at $t = 0$, so when the comet breaks up ($t = 18.75$ s), the ship is at $x = (18.75\text{ s})(0.6\,c) = 11.25\,c\text{s}$. The comet is 10 cs ahead of the ship, so it is at position $x = 11.25\,c\text{s} + 10\,c\text{s} = 21.25\,c\text{s}$.

ii) $x = 21.25\,c\text{s}$ $\quad x' = ?$

$t = 18.75\text{ s}$ $\quad t' = ?$

$V = 0.6\,c$

Our desired unknown is t', so we use

$$t' = \gamma\left(t - xV/c^2\right) = \tfrac{5}{4}\left((18.75\text{s}) - (21.25\,c\text{s})(0.6\,c)/c^2\right) = 7.5\text{ s}$$

Alternate method for part a, if you've already done part b:

Strategy: Use the Lorentz transform, with the event being the breakup of the comet.

Implementing the strategy:

$x = ?$ $\quad x' = 12.5\,c\text{s}$

$t = ?$ $\quad t' = 7.5\text{ s}$

$V = 0.6\,c$

Our desired unknown is t, so we use

$$t = \gamma\left(t' + x'V/c^2\right) = \tfrac{5}{4}\left((7.5\text{ s}) + (12.5\,c\text{s})(0.6\,c)/c^2\right) = 18.75\text{ s}$$