Physics 106b-2011 Final Exam Coverage

Notes

The exam will be a three hour take home. It will be available for pick-up from the secretaries' office on Monday; you will need to turn in a course evaluation form to the secretaries to receive your exam.

Coverage:

The exam <u>is</u> cumulative, and will include questions on material from the first part of the semester. (For example, there might be a question about Gauss's Law.) However, there will be extra emphasis on material covered since the last exam:

Last part of of class 23 (3-29-11) through end of course
Assignments 9-11
Wolfson & Pasachoff:
34-4 through 34-8
chapter 37 (omitting 37-4 and "Intensity in the Interference Pattern" on pp. 976-8)
Relativity Handouts ("Relativity Fundamentals Explained Well" and "Famous Paradoxes")
W&P 1024-1033

Critical concepts for material covered since exam 2

Electromagnetic Waves

- <u>What a plane wave is</u>
- Derivation (for planes waves) of the speed c and $E = cB$ from Ampère's law and Faraday's law
- <u>Understanding of the geometrical relationship between the direction of **E**, the direction of **B**, and the direction of propagation</u>
- <u>Relationships between λ, k, ω, f, T, and c.</u>
- Qualitative understanding of how em waves can be produced
- How em waves are detected (via the **E** component, the **B** component, or the intensity)
- <u>Fact that $\text{Intensity} \propto E^2$, meaning of Intensity (*i.e.* Power/Area)</u>
- For a small source, $S = \frac{P}{4\pi r^2}$
- Definition of Poynting vector, using it to find direction, intensity, and radiation pressure

Polarization

- Meaning of the term
- Be able to compute fraction of light passed through a series of polarizers
- <u>Be able to explain why inserting a polarizer between two crossed polarizers can increase the amount of light transmitted</u>
- Be able to discuss this both in wave picture and particle picture

Interference & Diffraction

- Huygen's principle
- <u>Condition for constructive interference (path lengths differ by $m\lambda$)</u>
- How to use this condition (e.g. to find peak positions for two-slit interference or diffraction grating)
- <u>Diffraction: $\theta_{first\ \min} \approx \frac{\lambda}{a}$</u>
- Rayleigh Criterion
- <u>How to find angular resolution of an instrument, given its aperture size</u>
- Meaning of diffraction order for a grating

Relativity

- <u>Why c has to be the same in all frames</u> (You should have a deep understanding of the theoretical reason for this from Maxwell's equations, and a superficial understanding of the experimental reason, *i.e.* the Michelson-Morley experiment.)

Derivation of time dilation from light clocks
Derivation of length contraction from time dilation
Derivation of synchronization from time dilation
How synchronization differences resolve the time dilation paradox
The Lorentz Transformation: how to use it
The Lorentz Velocity Transformation: how to use it
Relativistic mass, momentum, and energy
Rest energy

You are expected to be able to integrate and differentiate the following functions:

x^n (where *n* may be positive or negative, and may or may not be an integer), $\sin ax$, $\cos ax$, $\frac{1}{x}$, e^{ax}

Equations to internalize

(Not the format that they will be given in on the exam; for that, see the following pages.)

Ch. 23:

Coulomb's law: The force between two point charges: $\mathbf{F} = \frac{kq_1q_2}{r^2}\hat{\mathbf{r}}$

Relation between the electric force and the electric field: **F = E**q

Electric field of a point charge: $\mathbf{E} = \frac{kq}{r^2}\hat{\mathbf{r}}$ (This is really a variant form of Coulomb's law.)

Principle of superposition: $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \ldots$

Electric field of a continuous charge distribution: $\mathbf{E} = \int \frac{k\,dq}{r^2}\hat{\mathbf{r}}$ (Again, this is really a variant form of Coulomb's law.)

Ch. 24:

Definition of electric flux: $\phi_E \equiv \int \mathbf{E}\cdot\hat{\mathbf{n}}\,dA$ (for flat surface, uniform field, $\phi_E = \mathbf{E}\cdot\hat{\mathbf{n}}\,A$)

Gauss's law: $\oint \mathbf{E}\cdot\hat{\mathbf{n}}\,dA = \frac{q_{net\ enclosed}}{\varepsilon_o}$

Ch. 25:

Definition of voltage: $V \equiv \frac{U \text{ of a point charge}}{\text{charge of that point charge}}$

Relation of voltage to electric field: $\Delta V_{AB} = -\int_A^B \mathbf{E}\cdot d\vec{\ell}$

Voltage due to a point charge: $V = \frac{kq}{r}$

Superposition of voltages: $V_{TOT} = V_1 + V_2 + \ldots$

Voltage of a continuous charge distribution: $V = \int \frac{k\,dq}{r}$

Relation of electric field to voltage: $\mathbf{E} = -\vec{\nabla}V$, where $\vec{\nabla} \equiv \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$

(Note: variants of this would include, for example, $E_\ell = -\frac{dV}{d\ell}$.)

Definition of capacitance: $Q = CV$ $\quad \frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} \Leftrightarrow C_{series} = \frac{C_1C_2}{C_1 + C_2}$ $\quad C_{parallel} = C_1 + C_2$

Energy density of the electric field: $u_E = \frac{\varepsilon_0 E^2}{2}$ $\quad U = \int_{all\ space} u_E\ dV$

Dielectric constant: $\kappa \equiv \frac{E_{plates}}{E_{Tot}}$ $\quad C_{parallel\ plate} = \frac{\kappa\varepsilon_0 A}{d}$ $\quad U = \frac{1}{2}CV^2$

Ohm's Law: $V = IR$ $\quad \frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{r_2} \Leftrightarrow R_{parallel} = \frac{R_1R_2}{R_1 + R_2}$ $\quad R_{series} = R_1 + R_2$

Definition of resistivity: $R = \frac{\rho L}{A}$ **Resistivity in the Drude model:** $\rho = \frac{m}{ne^2\tau}$ **Drift velocity:** $I = nqAv_d$

Electrical power: $P = VI \quad P = I^2 R \quad P = V^2/R$ **Output voltage of a real battery:** $V_{out} = \varepsilon - Ir$

RC charging: $V_C = \varepsilon\left(1 - e^{-t/RC}\right)$ **RC discharging:** $V_C = V_0 e^{-t/RC}$

Hall voltage: $V_H = \frac{IB}{nqt}$ **Gauss's Law for magnetic fields:** $\oint \mathbf{B} \cdot \hat{\mathbf{n}}\, dA = 0$

Lorentz force: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

Force on a current-carrying wire: $d\mathbf{F} = I\, d\vec{\ell} \times \mathbf{B}$ **If wire is straight and B uniform:** $\mathbf{F} = I\, \vec{\ell} \times \mathbf{B}$

Biot-Savart Law: $d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{\mathbf{r}}}{r^2}$

Ampère's Law: $\oint \mathbf{B} \cdot d\vec{\ell} = \mu_0 I_{\substack{net \\ threading}} + \mu_0 \varepsilon_0 \frac{d}{dt}\int \mathbf{E} \cdot \hat{\mathbf{n}}\, dA \quad B_{solenoid} = \mu_0 nI$

Motional emf and/or induction: $\varepsilon_{loop} = -\frac{d\phi_B}{dt}$, where $\phi_B \equiv \int \mathbf{B} \cdot \hat{\mathbf{n}}\, dA$

Faraday's law: $\oint \mathbf{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int \mathbf{B} \cdot \hat{\mathbf{n}}\, dA$ **Transformers:** $\varepsilon_s = \varepsilon_p \frac{N_s}{N_p} \quad I_s V_s = I_p V_p$

Radiation: $E = cB \quad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \quad S = P_{rad}\, c$ **Compact source:** $S = \frac{P}{4\pi r^2}$

Interference: $m\lambda = d \sin\theta_{\max}$ **slit diffraction:** $\theta_{\min} = \frac{\lambda}{a}$ **hole diffraction:** $\theta_{\min} = 1.22\frac{\lambda}{a}$

Relativity: $V = \frac{\text{distance}}{\text{time}} \quad \gamma \equiv \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \quad \Delta t_{\text{other}} = \gamma\, \Delta t_{\text{proper}} \quad L_{\text{other}} = \frac{L_{\text{r}}}{\gamma} \quad \Delta t_{\text{synch}} = \frac{L_{\text{r}} V}{c^2}$

Lorentz transformation: $x = \gamma\left(x' + V t'\right) \quad t = \gamma\left(t' + x'V/c^2\right) \quad x' = \gamma\left(x - V t\right) \quad t' = \gamma\left(t - xV/c^2\right)$

Lorentz velocity transformation: $u_x = \frac{u'_x + V}{1 + u'_x V / c^2} \quad u_y = \frac{u'_y}{\gamma\left(1 + u'_x V / c^2\right)}$

Relativistic mass, momentum, & energy: $m_r = \gamma m_0 \quad p = m_r u \quad E_{\text{rest}} = m_0 c^2 \quad E_{\text{Tot}} = m_r c^2$

On the exam, you will be provided with the following numerical values:

$k = 8.99_9$ Nm^2/C^2	$\varepsilon_o = 8.85_{-12}$ C^2/Nm^2 = $1/(4\pi k)$	Charge of electron = -e, where e = 1.60_{-19} C
$c = 3.00_8$ m/s	$\mu_o = 4\pi_{-7}$ N/A^2	Mass of electron = $m_e = 9.11_{-31}$ kg
		Mass of proton = $m_p = 1.67_{-27}$ kg

Equations that will be provided on the exam
IMPORTANT: you are expected to know the conditions under which each of these can be applied!!!

$\mathbf{E} = \dfrac{\mathbf{F}}{q}$ $\mathbf{E}_{\substack{\text{point}\\\text{charge}}} = \dfrac{kq}{r^2}\hat{\mathbf{r}}$ $\mathbf{E} = \int \dfrac{k\,dq}{r^2}\hat{\mathbf{r}}$ $E_{\substack{\text{due to}\\\text{sheet}\\\text{charge}}} = \dfrac{\sigma}{2\varepsilon_0}$ $E_{\substack{\text{total,}\\\text{at metal}\\\text{surface}}} = \dfrac{\sigma}{\varepsilon_0}$

$\oint \mathbf{E}\cdot\hat{\mathbf{n}}\,dA = \dfrac{q_{\text{net, enclosed}}}{\varepsilon_0}$ $k = \dfrac{1}{4\pi\varepsilon_0}$ $\phi_E \equiv \int \mathbf{E}\cdot\hat{\mathbf{n}}\,dA$ $\phi_E = EA\cos\theta$

$V \equiv \dfrac{U_{\text{point}}}{q_{\text{point}}}$ $\Delta V_{AB} = -\int_A^B \mathbf{E}\cdot d\ell$ $\mathbf{E} = -\nabla V$ $\vec{\nabla} \equiv \hat{\mathbf{i}}\dfrac{\partial}{\partial x} + \hat{\mathbf{j}}\dfrac{\partial}{\partial y} + \hat{\mathbf{k}}\dfrac{\partial}{\partial z}$ $\Delta V_{AB} = -\mathbf{E}\cdot\ell$

$E = -\dfrac{dV}{dx}$

$V_{\substack{\text{point}\\\text{charge}}} = \dfrac{kq}{r}$ $V = \int \dfrac{k\,dq}{r}$

$v = v_0 + at$ $s = \Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2$ $v^2 = v_0^2 + 2as$ (and similar equations for y and z.)

$\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}$ $a_c = \dfrac{v^2}{r}$ $\mathbf{F}_{\text{net}} = \dfrac{d\mathbf{p}}{dt}$ $\mathbf{F}_{\text{net}} = m\mathbf{a}$

$F = -kx$ $\mathbf{W} = m\mathbf{g}$ $F_s \le \mu_s N$ $F_k = \mu_k N$ $F_c = \dfrac{mv^2}{r}$

$F_c = \dfrac{mv^2}{r}$ $W = \int_{\text{Start}}^{\text{Finish}} \mathbf{F}\cdot d\mathbf{s}$ $W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta$ $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$

$W_{ext} + Q_{ext} = \Delta K + \Delta U + \Delta U_{\text{int}}$ $W_{\text{int}} = \pm\Delta ?$ $U_{\text{grav}} = mgy$ $U_{\text{spring}} = \frac{1}{2}kx^2$ $K = \frac{1}{2}mv^2$

$\Delta U = -\int_{\text{Start}}^{\text{Finish}} \mathbf{F}\cdot d\mathbf{s}$ $\mathbf{F} = -\vec{\nabla}U$ $\vec{\nabla} \equiv \hat{\mathbf{i}}\dfrac{\partial}{\partial x} + \hat{\mathbf{j}}\dfrac{\partial}{\partial y} + \hat{\mathbf{k}}\dfrac{\partial}{\partial z}$ $F = -\dfrac{dU}{dx}$ $P \equiv \dfrac{dE}{dt}$

$P = \mathbf{F}\cdot\mathbf{v}$

$\mathbf{F} = -\dfrac{GMm}{r^2}\hat{\mathbf{r}}$ $U = -\dfrac{GMm}{r}$ $T^2 = \dfrac{4\pi^2 R^3}{GM}$ "Equal areas in equal times"

$\mathbf{R} = \dfrac{\sum m_i \mathbf{r}_i}{M_{\text{tot}}}$ $\mathbf{F}_{\text{net ext}} = M_{\text{tot}}\dfrac{d^2\mathbf{R}}{dt^2}$ $\mathbf{P} = \sum \mathbf{p}_i$ $\mathbf{F}_{\text{net ext}} = \dfrac{d\mathbf{P}}{dt}$ $v_{fA} = 2v_{CM} - v_{iA}$

$\text{Impulse} = \mathbf{F}_{\text{av}}\,\Delta t = \Delta\mathbf{P}$

$\omega \equiv \dfrac{d\theta}{dt}$ $\boldsymbol{\alpha} = d\boldsymbol{\omega}/dt$ $v_t = \omega r$ $a_t = \alpha r$ $\omega = \omega_0 + \alpha t$ $\Delta\theta = \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta$

$\mathbf{A}\times\mathbf{B} \equiv AB\sin\theta\,\hat{\mathbf{n}}$ $I = \sum m_i r_i^2$ $I = I_{\text{CM}} + Mh^2$ $\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$

$K = \frac{1}{2}mv_{\text{CM}}^2 + K_{\text{rot}}$ $K_{\text{rot}} = \frac{1}{2}I\omega^2$ $W = \tau\,\Delta\theta$ $v_{\text{CM}} = \omega r$ $\mathbf{L} = I\,\boldsymbol{\omega} = \mathbf{r}\times\mathbf{p}$ $\boldsymbol{\tau}_{\text{net ext}} = d\mathbf{L}/dt$

$\omega_0 = \sqrt{\dfrac{k}{m}}$ $\omega = \sqrt{\dfrac{g}{\ell}}$ $\omega = \dfrac{2\pi}{T} = 2\pi f$ $x = A\cos(\omega t + \varphi)$ $\gamma \equiv \dfrac{b}{m}$ $Q = \dfrac{\omega_s}{\gamma}$

$$y = A\cos(kx \pm \omega t) \qquad k = \frac{2\pi}{\lambda} \qquad v = \frac{\lambda}{T} = \frac{\omega}{k} \qquad I \equiv \frac{P}{A} \qquad I \equiv \frac{P}{4\pi r^2}$$

$$y = A\cos(kx + \omega t) + A\cos(kx + \omega t) = 2A\cos\omega t \sin kx \qquad L = \frac{n}{2}\lambda$$

$P \equiv \frac{F}{A}$ The equipartition theorem. $P = \frac{kA\Delta T}{\Delta x}$ $R \equiv \frac{\Delta x}{k}$ $P = e\sigma AT^4$

$$PV = Nk_B T = nRT \quad C \equiv \frac{dQ}{dT} \quad c \equiv \frac{C}{m} \quad C_V = \frac{dU_{INT}}{dT} \quad C_m \equiv \frac{C}{n} \quad C_{mV,\text{solid}} = 3R$$

$$Q = CV \qquad \frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} \Leftrightarrow C_{series} = \frac{C_1 C_2}{C_1 + C_2} \quad C_{parallel} = C_1 + C_2$$

$$u_E = \frac{\varepsilon_0 E^2}{2} \qquad U = \int_{\text{all space}} u_E \, dV \qquad \kappa \equiv \frac{E_{plates}}{E_{Tot}} \qquad C_{\substack{parallel\\plate}} = \frac{\kappa \varepsilon_0 A}{d} \qquad U = \tfrac{1}{2} CV^2$$

$$V = IR \qquad \frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{r_2} \Leftrightarrow R_{parallel} = \frac{R_1 R_2}{R_1 + R_2} \qquad R_{series} = R_1 + R_2$$

$$R = \frac{\rho L}{A} \qquad \rho = \frac{m}{ne^2\tau} \qquad I = nqAv_d \quad P = VI \qquad P = I^2 R \qquad P = V^2/R \qquad V_{out} = \varepsilon - Ir$$

$$V_C = \varepsilon\left(1 - e^{-t/RC}\right) \quad V_C = V_0 e^{-t/RC} \quad V_H = \frac{IB}{nqt} \qquad \oint \mathbf{B}\cdot\hat{\mathbf{n}}\, dA = 0 \quad \mathbf{F} = q\mathbf{v}\times\mathbf{B}$$

$$d\mathbf{F} = I\, d\vec{\ell} \times \mathbf{B} \qquad \mathbf{F} = I\, \vec{\ell} \times \mathbf{B} \qquad d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\ell} \times \hat{\mathbf{r}}}{r^2}$$

$$\oint \mathbf{B}\cdot d\vec{\ell} = \mu_0 I_{\substack{net\\threading}} + \mu_0 \varepsilon_0 \frac{d}{dt}\int \mathbf{E}\cdot\hat{\mathbf{n}}\, dA \qquad B_{solenoid} = \mu_0 n I$$

$$\varepsilon_{loop} = -\frac{d\phi_B}{dt} \qquad \phi_B \equiv \int \mathbf{B}\cdot\hat{\mathbf{n}}\, dA \quad \oint \mathbf{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int \mathbf{B}\cdot\hat{\mathbf{n}}\, dA \qquad \varepsilon_s = \varepsilon_p \frac{N_s}{N_p} \quad I_s V_s = I_p V_p$$

$$E = cB \quad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \mathbf{S} = \frac{\mathbf{E}\times\mathbf{B}}{\mu_0} \quad S = P_{rad}\, c \quad S = \frac{P}{4\pi r^2}$$

$$m\lambda = d\sin\theta_{\max} \quad \theta_{\min} = \frac{\lambda}{a} \qquad \theta_{\min} = 1.22\frac{\lambda}{a}$$

$$V = \frac{\text{distance}}{\text{time}} \quad \gamma \equiv \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \quad \Delta t_{\text{other}} = \gamma\,\Delta t_{\text{proper}} \quad L_{\text{other}} = \frac{L_{\text{r}}}{\gamma} \quad \Delta t_{\text{synch}} = \frac{L_{\text{r}}\, V}{c^2}$$

$$x = \gamma(x' + V t') \quad t = \gamma\left(t' + x'V/c^2\right) \quad x' = \gamma(x - V t) \quad t' = \gamma\left(t - xV/c^2\right)$$

$$u_x = \frac{u'_x + V}{1 + u'_x V/c^2} \quad u_y = \frac{u'_y}{\gamma\left(1 + u'_x V/c^2\right)}$$

$$m_r = \gamma m_0 \qquad p = m_r u \qquad E_{\text{rest}} = m_0 c^2 \qquad E_{\text{Tot}} = m_r c^2$$