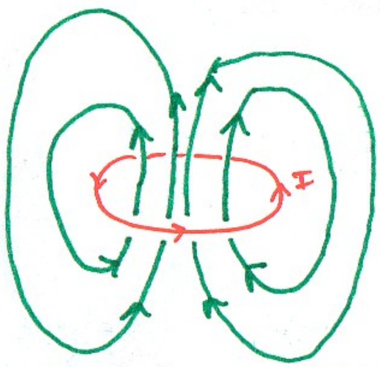
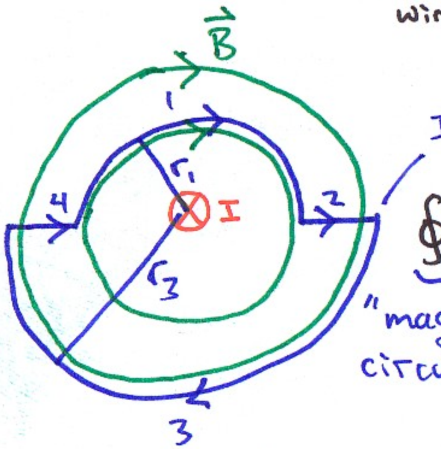
Model for permanent magnets



The field due to a current loop looks the same as that of a permanent magnet. It is now well established that permanent magnetism is due to the tiny "current loops" associated with electron spin.

Ampère's Law

Book: Biot-Savart $\dashrightarrow B_{\substack{\infty \\ \text{wire}}} = \dfrac{\mu_0 I}{2\pi r}$



$$\underbrace{\oint \vec{B}\cdot d\vec{l}}_{\text{"magnetic circulation"}} = \int_1 + \underbrace{\int_2 + \int_3 + \int_4}$$

$=0$ because $\vec{B} \perp d\vec{l}$

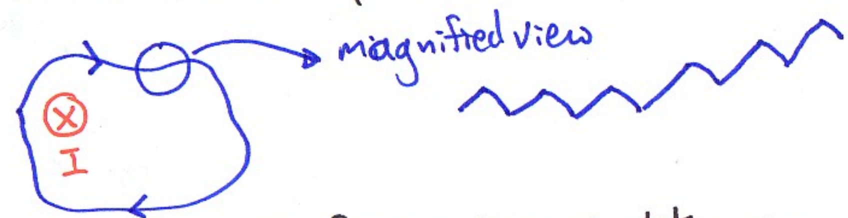
For 1 & 3, $\vec{B} \parallel d\vec{l} \Rightarrow \oint \vec{B}\cdot d\vec{l} = \int_1 B\,dl + \int_3 B\,dl$

B is constant along 1 & 3

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi r_1}\underbrace{\int_1 dl}_{\frac{2\pi r_1}{2}} + \frac{\mu_0 I}{2\pi r_3}\underbrace{\int_3 dl}_{\frac{2\pi r_3}{2}}$$

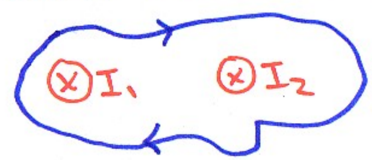
$= \mu_0 I$ Note: no r-dependence!

We can make any loop by combining radial & circumferential pieces:



$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I$ for any loop around the wire

What if there are two wires?



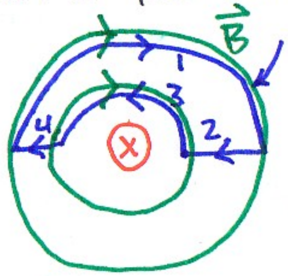
$\vec{B}_{TOT} = \underbrace{\vec{B}_1}_{\vec{B}\text{ due to } I_1} + \underbrace{\vec{B}_2}_{\vec{B}\text{ due to } I_2}$

$$\oint \vec{B}_{TOT}\cdot d\vec{l} = \oint (\vec{B}_1 + \vec{B}_2)\cdot d\vec{l} = \oint \vec{B}_1\cdot d\vec{l} + \oint \vec{B}_2\cdot d\vec{l}$$

$$= \mu_0 I_1 + \mu_0 I_2$$

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{net}$$

What if the current doesn't thread the loop?



Along 1, $\vec{B} \parallel d\vec{l} \Rightarrow \int_1 \vec{B}\cdot d\vec{l} = \int_1 B\,dl = \frac{\mu_0 I}{2}$

Along 3, $\underbrace{\vec{B} \uparrow\downarrow d\vec{l}}_{\text{anti-parallel}} \Rightarrow \int_3 \vec{B}\cdot d\vec{l} = -\int_3 B\,dl = \frac{-\mu_0 I}{2}$

$\Rightarrow \oint \vec{B}\cdot d\vec{l} = 0 \Rightarrow I$ only "counts" toward $\oint \vec{B}\cdot d\vec{l}$ if it threads the loop.

Putting this all together:

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\substack{net \\ \text{threading}}} + \text{term that depends on changing } \vec{E}}$$

Ampère's Law: The third MEQ