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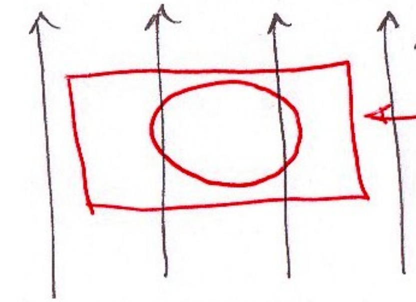
Practice w/ circulation

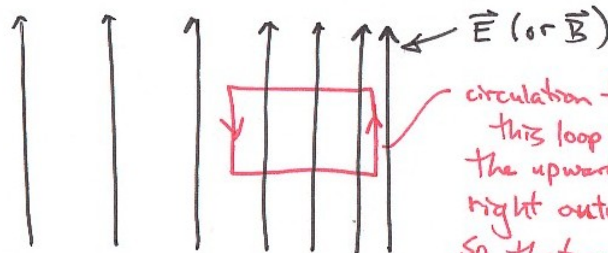
Recall $\oint \vec{B} \cdot d\vec{l}$ is the "circulation of $\vec{B}$"

** The circulation of $\left\{ \begin{matrix} \vec{B} \\ \vec{E} \end{matrix} \right\} > 0$ if a $\left\{ \begin{matrix} N \text{ pole} \\ + \text{ charge} \end{matrix} \right\}$ would get a net push around the mathematical loop in the direction of $d\vec{l}$. **

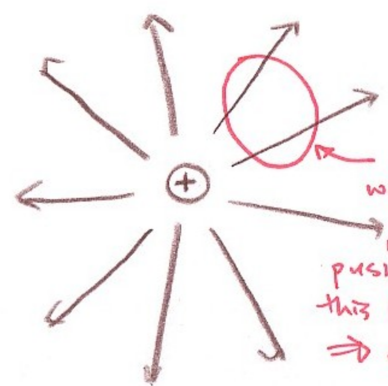
Example



no net push around either path $\Rightarrow$ circulation $= 0$ for either path



circulation for this loop is positive -- the upward push on the right outweighs that on the left, so that a $\oplus$ (or an N pole) would get a net ccl push around the loop



a $\oplus$ would experience no net push around this loop $\Rightarrow$ circulation $= 0$

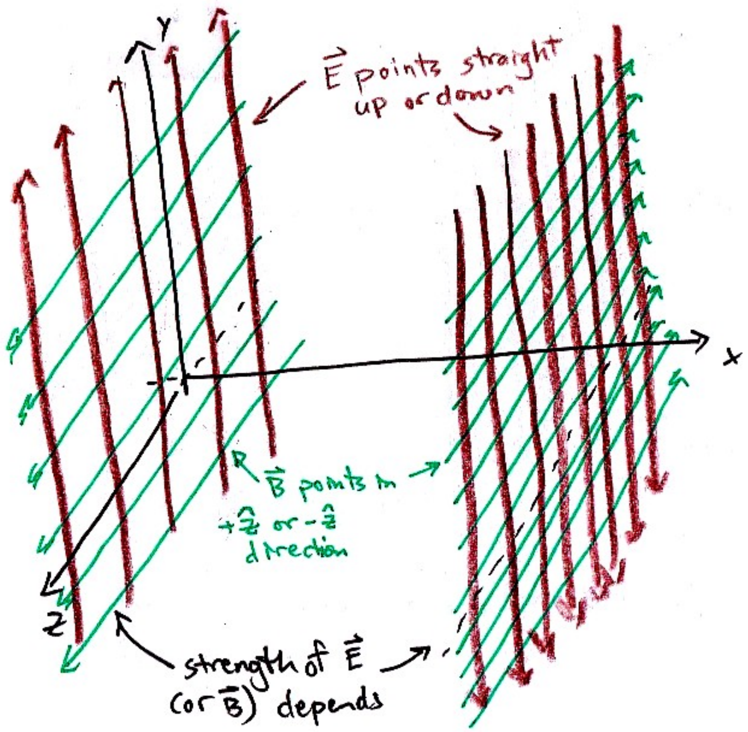
By comparing this example with the uniform field, we see that only non-uniform fields can have non-zero circulation. But, not all non-uniform fields have non zero circulation

Electromagnetic Radiation

To start, we ask whether the following configuration of electromagnetic fields is allowed in vacuum ($Q=0$, $I=0$) and if so, are there some restrictions, e.g. on the relative amplitudes of $\vec{E}$ and $\vec{B}$:

Is this allowed by MEQ's in vacuum?

$$\left\{ \begin{array}{l} \vec{E}(x,y,z,t) = E(x,t)\,\hat{y} \\ \vec{B}(x,y,z,t) = B(x,t)\,\hat{z} \end{array} \right\} \text{"plane wave"}$$



... but for all points in a plane $\perp$ to $\hat{x}$, $\vec{E}$ (or $\vec{B}$) is uniform.

It's easy to see that this plane wave satisfies Gauss's law & no monopoles law: any flux entering a Gaussian box also exits it.

Does it satisfy Ampère's Law & Faraday's Law?

Do plane waves in vacuum satisfy MEQs? (ctd.)

Faraday's law : $\underbrace{\oint \vec{E} \cdot d\vec{l}}_{\text{as we've seen, this is related to the spatial variation of } \vec{E}} = \underbrace{-\frac{d}{dt}\int \vec{B} \cdot \hat{n}\, dA}_{\text{obviously this is related to the time variation of } \vec{B}}$

$\vec{E} = E(x,t)\,\hat{y}$

$\vec{B} = B(x,t)\,\hat{z}$

$\xrightarrow[\text{pp. 890-1}]{\text{Book}} \quad \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \quad (1)$

Ampère's Law in vacuum $(I=0)$: $\underbrace{\oint \vec{B} \cdot d\vec{l}}_{\text{related to spatial variation of } \vec{B}} = \underbrace{\mu_0 \varepsilon_0 \frac{d}{dt}\int \vec{E} \cdot \hat{n}\, dA}_{\text{related to time variation of } \vec{E}}$

$\xrightarrow[\text{pp. 890-1}]{\text{Book}} \quad \frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \quad (2)$

(1) & (2) are two coupled DEQs. Let's "guess" a solution, then plug it into them to see if the guess works (i.e. if it satisfies the equations), and if there are restrictions required (e.g. on amplitudes of E & B) to make it work. Here's our guess, a "sinusoidal plane wave": *

$$\boxed{\begin{aligned}\vec{E} &= E_p \sin(kx - \omega t)\,\hat{y} \\ \vec{B} &= B_p \sin(kx - \omega t)\,\hat{z}\end{aligned}}\ *$$