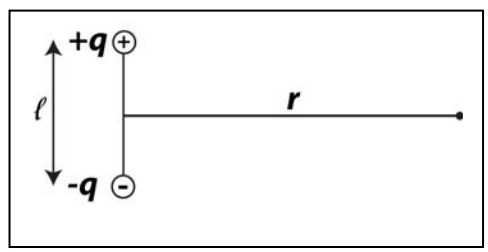
Physics 102 Spring 2012 Problem Set 3 (Corrected) due Fri. Feb. 10, 4 pm

Reading: Chapter 17

1) **Linear distributions of Charge & DNA.** (a) DNA is a linear molecule with one negative elementary charge per base. (The negative charge is actually on the phosphate group in the backbone, but for this problem you just need to know that there is one negative charge per base.) The spacing between bases along the axis of the DNA double helix is 3.4 Å; of course, the bases come in pairs, so there are two negative charges per 3.4 Å of helix length. (1 Å = 10^{-10} m) In this problem, approximate the DNA as a uniformly-charged rod. Find the electric force on a positively charged ion with charge + e at a distance 0.500 nm from the center of a very long DNA molecule. Use K = 76.5 for the dielectric constant of the surrounding medium. (b) Now, compute the change in potential energy when a positive elementary charge on an ion moves from far away (say, R = 1 micron away = 10^{-6} m) to 0.5 nm (near the DNA's surface). Compare this energy to thermal energy at room temperature, $\frac{1}{2}k_bT$. Is the ion likely to stay in the vicinity of the DNA molecule or wander off due to its thermal energy?

2) **Electric potential energy and potential** Suppose we have three ions arranged in a line as shown below. They are chlorine (Cl^-), sodium (Na^+) and calcium (Ca^{2+}); each has charges that are multiples of e as indicated. **Find the electrical potential energy** of the sodium ion and **the electrical potential (i.e. the voltage) at the position of the sodium ion** due to the interaction with other two ions, assuming they are in water with dielectric constant = 80.0

3) **Field of a dipole.** Your text shows that the electric field due to a dipole has a strength $E = \frac{2kp}{r^3}$ for points along the axis of the dipole that are at a distance $r >> \ell$, where ℓ is the distance between the two charges of the dipole. Show that, for points along an axis perpendicular to the dipole that are at a distance $r >> \ell$ (as shown in the figure), the field is $E = \frac{kp}{r^3}$, i.e. half as strong as for points along the dipole axis.



4) **Force on a dipole.** A dipole with moment p is a distance $r >> \ell$ away from a point charge q, where ℓ is the distance between the two charges of the dipole. The axis of the dipole is oriented radially away from the point charge. What is the magnitude of the force on the dipole in terms of p, r, and q?

5) **Electrostatic interaction energies:** This looks awfully messy, but isn't because you are using scalars and you can use a spreadsheet to do the gruntwork.

The double helix of DNA is held together like a zipper by electrostatic interactions between chemical groups on each of its helices that meet and pair up in the helices' center. These "basepairs" run down the center of the double helix, as shown below. The chemical groups that pair up include thymine, adenine, cytosine and guanine, with chemical structures shown below. Solid lines mean chemical bonds, C is carbon, O is oxygen, N is nitrogen and H hydrogen. Each of these groups includes atoms that are charged under biologically relevant conditions; some values are given below. Dotted lines indicate "hydrogen bonding" an important form of electrostatic interaction for biology, although in fact all of the charges exert forces on each other. You can see from the figures that the O in Thymine is part of an electric dipole with a carbon, while both Thymine and Adenine have N-H dipoles.

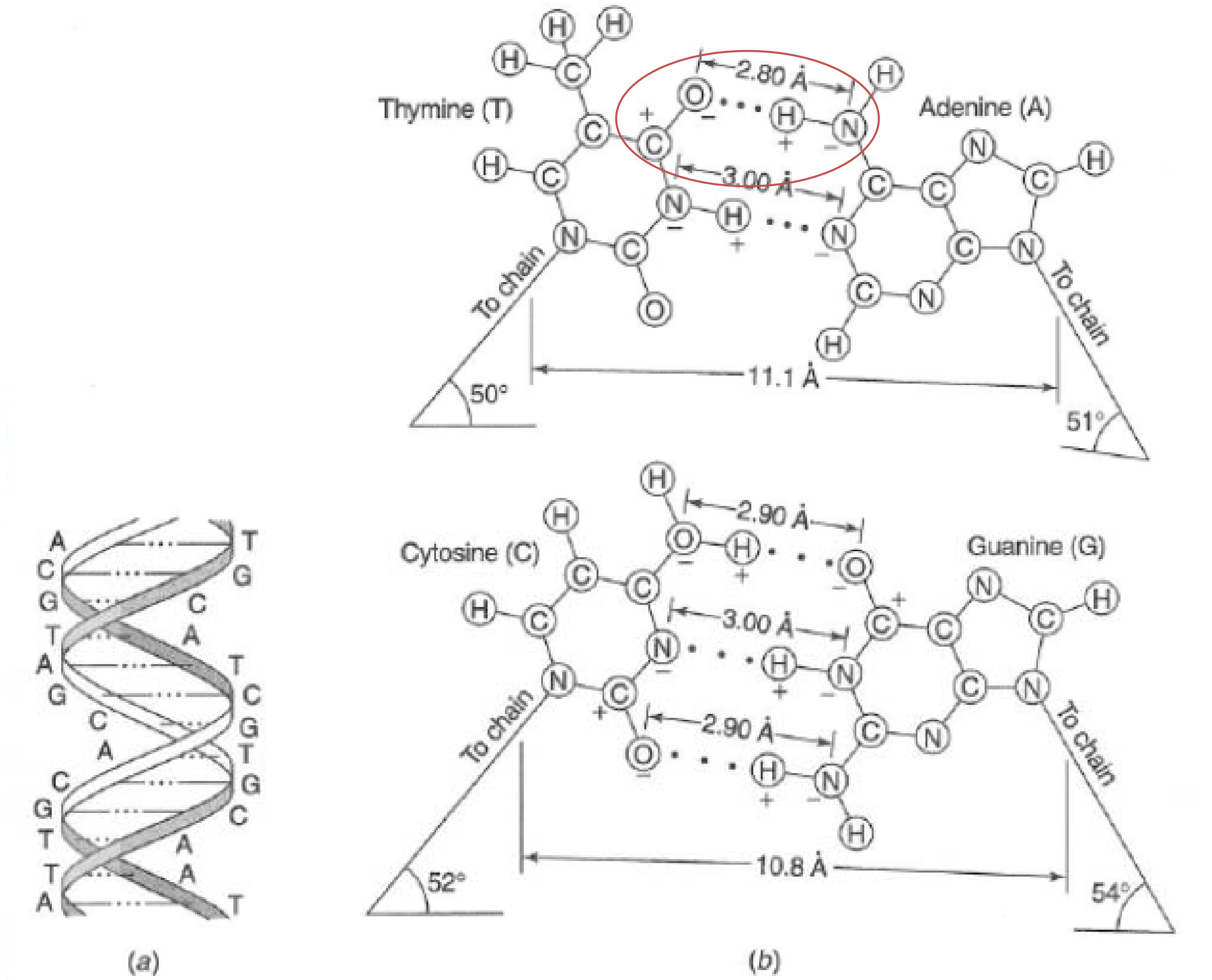


FIGURE 18.1 (*a*) Section of DNA double helix and (*b*) close-up view showing how A and T and C and G are always paired (the distance unit is 1 Å = 10^{-10} m).

Calculate the electrostatic interaction potential energy between the C=O dipole of thymine and the H-N dipole of adenine (circled), assuming that r_{CH} = 0.31 nm, r_{CN} = 0.41 nm, r_{OH} = 0.19 nm and r_{ON} = 0.29 nm. The charges involved are $q_H = -q_N = 0.19\,e$ and $q_C = -q_O = 0.41\,e$. **You can treat each charge as an individual point charge that interacts with *every other charge* of interest; there's no need to treat them as dipoles!)** Assume a dielectric constant $K = 1$ because this interaction takes place in the middle of the DNA, where there is no room for water molecules. **It's OK to do this by showing how you'd calculate one of these interactions by hand, and doing the rest (on your own—no copying!) by spreadsheet,** explaining how you did so carefully. Compare your value to thermal energy $\frac{1}{2}k_bT$ at body temperature. Is there enough thermal energy available to disrupt DNA base pairing?

6) **(Independent problem) Capacitors: charge, E-field & breakdown** Show all your work. (a) Students in sophomore physics lab apply 12 V across their 2.5 pF ceramic capacitor. What is the charge on each electrode of the capacitor? (b) The capacitor has a gap spacing of 5.0 microns. What is the electric field in this capacitor? (c) How large a voltage can they apply before it exceeds the dielectric strength of 200 V per 0.001 inch thickness in their capacitor?

7) **(Independent problem) Capacitors & defibrillation** A medical defibrillator stores energy in a giant capacitor so that this energy can be discharged through the heart of a person undergoing fibrillation—a condition where the heart begins to beat in a random, uncoordinated manner— with the goal of restoring their heart's natural rhythm. A medical defibrillator stores 950 J of energy in a 100 μF capacitor. (a) What is the voltage across the capacitor? (b) If the capacitopr discharges 300 J of its energy in 2.5 ms, what is the power delivered to the patient? (As they say when using this device, STAND BACK!)

8) **Energy and even bigger capacitors**: We live inside a giant capacitor! Its plates are the Earth's surface and the ionosphere, a conducting layer of the atmosphere beginning at about 60 km altitude. (a) What is its capacitance (you may treat it approximately as a parallel plate capacitor, even though it's really spherical—can you see why?) (b) The potential difference between the Earth and the ionosphere is typically about 6 MV! Find the total energy stored in our planetary capacitor!

9) **Capacitances in nature** Explain how you get each answer.
(a) A cell membrane has dielectric constant $K = 5.00$ and thickness $\ell = 7.50$ nm. Its outer surface is charged positively and its inner region is negatively charged. Assuming its area is 5.00×10^{-9} m^2, **what is its effective capacitance**? (You may approximate the cell membrane as a parallel plate capacitance throughout this problem.)
(b) If the potential on the outer surface of the membrane is +60 mV greater than on the inside surface, **how much total charge** resides on the outer surface? Give your answer in Coulombs, then as a multiple of elementary charge, *e*.
(c) **How much electrical energy is stored** in the cell as a result of this *static* electric field? (d) In nervous conduction, the electrical potential shifts from approximately -70 mV across the membrane of the nerve cells called axons to +30 mV and then back again in a pulse called an action potential. This pulse propagates down the axon, carrying the nerve signal. (Like so much else, this picture is both accurate as far as it goes and a bit oversimplified. The action potential entails the flow of ions across the membrane, so it's more complicated than simpl electrical circuits!)
How much energy is required to recharge the membrane by this approximately 100 mV potential difference? Assume the axon is 1m long and 5 micron (10^{-6} m) in radius, with a capacitance per unit area of 0.01 F/m^2. Again, model this situation simply as a parallel plate capacitor.