Physics 102 Spring 2012 Problem Set 2 (Revised) due Fri. Feb. 3, 4 pm

Reading: Chapter 16

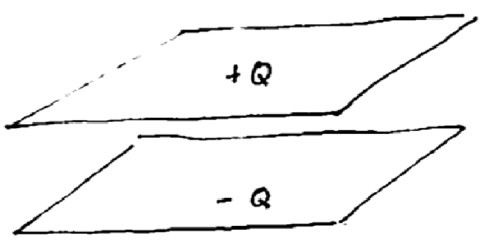
1) (Independent problem, written by Prof. Catherine Crouch) A uniformly charged sheet measures 1.00 m on each side and the total charge on it is 1.00 µC. Find the field strength (a) 1.00 cm from the center of the sheet and (b) 1.00 km from the center of the sheet. In each case explain clearly what approximation you are making and why that approximation is reasonable.

2) (Independent problem) A solid sphere of radius R has a charge -Q spread uniformly through its volume. a) Sketch the electric field lines for this situation, using the scheme where the density of lines indicates the strength of the field. Your sketch should indicate what happens to the lines both inside and outside the sphere. b) Draw a graph of E as a function of radius r, from $r = 0$ to $r = 3R$. Label the vertical axis of your graph quantitatively.

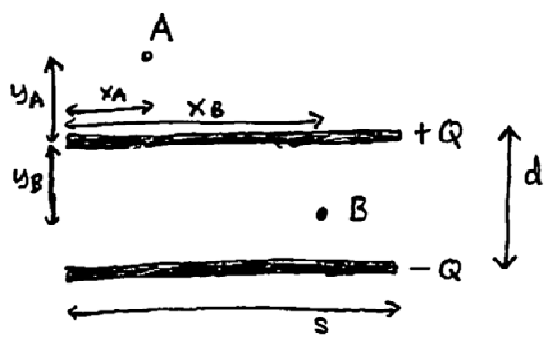
3) Sternheim & Kane problem 16-72

4) Sternheim & Kane problem 16-76. You are meant to assume that the ring is very thin, i.e. that the inner radius and outer radius are almost the same.

5) (written by Prof. Catherine Crouch) Two square conducting plates of side length s are given opposite charges $+Q$ and $-Q$ as shown in the diagram. The diagram is not to scale; points A and B are both near the middle of the plates, far from the edges, and the spacing between plates d is much smaller than the side lengh s.



Perspective view



Front view

(*a*) Is the potential difference from point A to point B, $\Delta V_{AB} \equiv V_B - V_A$, positive, negative, or zero? Explain briefly.

(*b*) Find $\Delta V_{AB} \equiv V_B - V_A$ in terms of Q and the distances defined on the diagram (all of which are positive numbers), and fundamental constants. Explain the logic behind your calculation briefly.

6) **Electrical potentials:** (a) A potential difference of 0.0900 V exists across a cell membrane. The thickness of the membrane is 8.50 nm. What is the magnitude of the electric field across the membrane? (b) What force does one sodium ion (written as Na^+, and having charge $+e$) experience as a result of this electric field when in the membrane? (c) How much energy is required for a sodium-potassium pump (the enzyme that transfers Na^+ ions from the inside of the cell to the outside, and K^+ from the outside to the inside) to transport three Na^+ ions from the inside to the outside of the cell?

7) (written by Prof. Catherine Crouch) One sodium chloride (NaCl) "molecule" consists of a Na^+ ion of charge $+e$ and a Cl^- ion of charge $-e$ separated by a distance of 0.24 nm. For this problem, assume that the ions are embedded in the same lipid material as a cell membrane (K = 8.00). (*a*) Find the amount of work by an external force required to remove one ion to infinitely far away from the other of this molecule. This quantity is sometimes called the binding energy of a molecule. Compare to the binding energy in air (K = 1.00) and in water (K = 80.0). (*b*) Suppose the charges of the Na^+ and Cl^- were doubled to +2e and -2e respectively. By what factor would the binding energy change? Explain briefly.

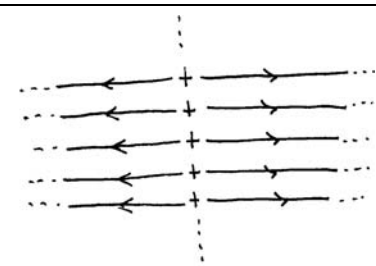
8) Electrostatics & biochemistry grab bag:

The relevant values of dielectric constant, K are 1 for vacuum, ~3 for oils, ~8 for a cell membrane, and about 80 for water. (Here ~ or tilde means "approximately equal to".)

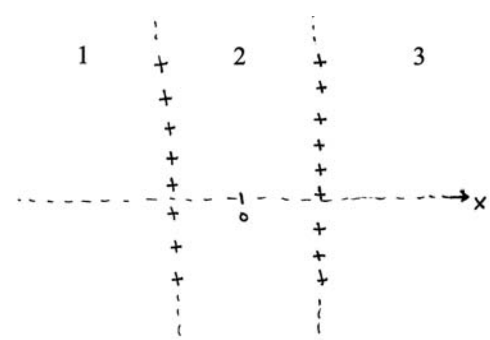
(a) Calculate the ***magnitude* of force** between two elementary electronic charges, e, separated by 1.00 nm (molecular dimensions) in vacuum, oil and water. **How much greater or smaller are these forces than the ~ 6 pN forces** we see are often provided by molecular motor proteins (the little chemical machines that move things around in cells.) (Comment: Molecules have dimensions on the nanometer scale, and often have patches of charge about this size on their surface, so this comparison is a very important one for biochemistry.)

(b) The energy available in the environment of our proteins due to just thermal energy ½ $k_b\, T$ (where k_b is Boltzmann's constant and T is in units of Kelvin, those of absolute temperature.). Compute the "Bjerrum length", l_B, defined as the distance between two elementary charges e where their electrical potential energy equals this thermal energy, assuming T = 300 K. Assume a dielectric constant K = 76.5 for the cell's interior. (This length tells you over what distances thermal energy can overcome electrostatic interactions.)

9) As we have discussed, the magnitude of the electric field due to an infinite thin sheet (*i.e.* an infinite plane) of charge is $E = 2\pi k\sigma$, where σ (Greek letter sigma) is the charge per unit area (and has units of C/m^2). As you can see from the formula, the field strength does not depend on distance away from the sheet. This might seem surprising, but perhaps it helps to think how the sheet would look to you as you moved away from it – no matter how far away you moved, it would look just the same – like an infinite sheet! It might also surprise you that the field isn't infinite – after all, the total charge on the sheet is infinite. However, because the field due to an infinitesimal chunk of the sheet falls off as $1/r^2$, the total field is finite, as shown by your book.



Cross-sectional view of an infinite sheet of charge and the field it creates.



The field points perpendicular to the sheet (straight away from the sheet if $\sigma > 0$, or straight toward the sheet if $\sigma < 0$), as shown in the top figure. The lower figure shows a cross-sectional view of two infinite sheets. The x-axis is shown as a horizontal line, with the origin centered between the two sheets. The charge density σ is the same for both sheets.

a. Using the "length of vector indicates strength of field" graphical scheme, sketch the electric field due to the left sheet (label it "E_L"), the electric field due to the right sheet (label it "E_R"), and the total electric field (label it "E_{TOT}") in each of the three regions 1, 2, and 3. Because the field has such a simple form, a single vector for each of the three fields for each of the three regions is fine, making a total of nine vectors.

b. As for potential energy, we are free to define the zero of voltage at any position we choose – only <u>changes</u> in voltage are physically significant. Let's define $V = 0$ at $x = 0$. Sketch qualitatively the voltage along the x-axis. On your sketch, indicate the positions of the two sheets of charge.

c. Identify a pair of points on the x-axis, point A and point B, for which point A has a higher voltage but lower E than point B. (This emphasizes that the voltage is not proportional to the field, but instead depends on the line integral of the field.)