A Brief Discussion of Unit Systems

Introduction

Physics students usually begin their study of the discipline using the MKS or SI (Systém International) unit system, based on the familiar metric standards of meter for length, kilogram for mass and seconds for time. Students easily become familiar with conversion of quantities expressed in standard MKS units to those in other unit systems. For example 1 eV ('electron volt') = 1.602^{-19} J ('joule'), or 1 btu ('British thermal unit') = 1055 J; etc. Simple conversion factors are all that is needed to manipulate these quantities.

Later, usually in the first treatment of quantum mechanics or electricity and magnetism, the CGS or Gaussian unit system, based on the centimeter (cm) for length, gram (g) for mass and seconds (sec) for time, is often introduced. Initially this looks like no problem. Conversions apparently can be done as above. For example, the energy unit, the erg = g $cm^2/sec^2 = 10^{-7}$ J as expected from the separate conversion factors 1 g = 10^{-3} kg and 1 cm = 10^{-2} m. All is well until one discovers that, the changes actually go further than simple conversions. Some of the basic laws of physics actually change form! For example, Coulomb's law looks different in the two systems.

MKS CGS

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} \qquad\qquad F = \frac{q_1 q_2}{r^2} \qquad (1)$$

where q_1 and q_2 are electric charges and r is the distance between the charges. In the MKS system charges are measured in coulombs (C). The constant ε_o that appears in the MKS definition is called the permittivity of free space and has the value of $8.85\text{x}10^{-12}\ C^2/Nm^2$. In the CGS system, on the other hand, there is no constant of proportionality.. It would appear that charge should be given in units of $\sqrt{dyne}$ cm where the dyne is the force unit in the CGS system (dyne = gm cm/sec^2). What is going on here? Charge seems to be quantified in different ways in these two unit systems. This Appendix addresses this situation. It is based on Appendices found in two standard physics textbooks:

Electricity and Magnetism by Edward M. Purcell (Vol. 2 of the Berkeley Physics Course);

Classical Electrodynamics by J. D. Jackson.

The student is referred to those sources and references therein for further information.

Explanation

To examine this issue we must deal with electricity and magnetism in a unified way. There are two basic and related electromagnetic forces, an electric force between two charges, or between a charge and an electric field (typified by Coulomb's law above)

$$F_E = k_E \frac{q_1 q_2}{r^2} \tag{2}$$

and a magnetic force acting between currents, or between a current and a magnetic field (as in the standard expression for the force per unit length between current carrying wires)

$$\frac{F_M}{L} = 2k_M \frac{I_1 I_2}{r} \ . \tag{3}$$

(Eqs.(2,3) describe these forces independently of unit system; expressing only the dependencies on current, charge and distance.) Obviously the values of k_E and k_M depend on the units in which charge or current are measured. (In all unit systems currents are measured in units of charge per second).

The first key point to note is that the ratio k_E/k_M is fixed by relativistic considerations[3] and has the value c^2 where c is the speed of light. (See the derivation in Purcell's appendix)

If we now presume that the units of force and length have already been selected and that currents will be measured in units of charge units per second, we still have one choice to make. We can, for example, choose a value for k_E. This will determine everything--k_M is required to be k_E/c^2 and the units in which charge will be measured will also have been thereby determined. This is the route taken in the CGS system. k_E is chosen to have the value of unity without dimension. Obviously then $k_M = 1/c^2$ and the units of charge will be $\sqrt{\text{dyne}}$ cm = $g^{1/2}cm^{3/2}/sec$. (This unwieldly

[3]Even if you haven't studied relativity in connection with electric and magnetic fields you can deduce that there must be important connections from the following observation. If a charge is stationary in one reference frame transforming to another reference frame puts the charge in motion so that it becomes a current. A stationary charge produces an electric field and a current gives rise to a magnetic field, so what looks like an electric field to one observer, can look like a magnetic field to another observer moving relative to the first.

combination is called for convenience sake the 'esu', for electrostatic unit in the CGS system) Currents are then measured in units of esu / sec = $\sqrt{\text{dyne}}$ cm / sec = $g^{1/2}cm^{3/2}/sec^2$. Eqs. (2, 3) therefore have the forms

$$F_E = \frac{q_1 q_2}{r^2} \quad ; \qquad \frac{F_M}{L} = \frac{2I_1 I_2}{c^2 r}$$

in the CGS system. This choice is convenient for theoretical calculations since one fewer constant is needed and the relativistic connection between electricity and magnetism is manifest.

In the MKS system, on the other hand, an entirely different approach is taken. Instead of choosing a value for either k, a new unit, the Coulomb (C), is defined for charge--a unit of convenient scale for engineering purposes[4]. Having chosen a unit for charge, k_E and k_M are fixed by eqs. (2,3). Through the definition of the amp outlined above k_M will turn out to be 10^{-7} N / A^2; and the MKS value of k_E is in turn $c^2 k_M$ so that

$$k_E = (2.9979 \times 10^8 \text{ m/sec})^2\, 10^{-7} \text{ N/A}^2$$
$$= 8.9874 \times 10^9 \text{ N m}^2/(\text{A sec})^2$$
$$= 8.9874 \times 10^9 \text{ N m}^2/\text{Coul}^2$$
$$= \frac{1}{4\pi\varepsilon_o}$$

We conclude by illustrating some simple traps to avoid and giving a set of instructions for converting quantities between the unit systems.

What can go wrong?

The biggest danger follows from the fact that there is no fundamental unit of charge in the CGS system. Based on eg. (1b) one could consistently measure charge in units of $\sqrt{\text{dyne}}$ cm = $\sqrt{\text{gm cm}^3}$ / sec. Electric fields, defined through the force per unit charge, **F** = q**E**, or via their connection with electric potential difference, **E** = V, have the odd units of dyne / ($\sqrt{\text{dyne}}$ cm) = $\sqrt{\text{dyne}}$ /cm which in turn requires V to be measured in units of $\sqrt{\text{dyne}}$. Yet V^2 is

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[4]The coulomb is defined to be 1 amp sec = 6.24×10^{18} electrons sec so that it is really the ampere (A) that is being defined as the 4th fundamental unit in the SI system. This amounts to the same thing as choosing a unit for charge, of course. The definition of the ampere or amp is actually through eq. (3). The amp is that current which when flowing in each of two infinitely long parallel wires of negligible cross-sectional area, separated by a distance of l meter in vacuum, causes a transverse force per unit length of 2×10^{-7} N / m to act between the wires.

certainly not a force! All this very much hinders the transparent connection between dimensions of a quantity and the forms of those laws in which it figures. In particular the units have no way of telling us how many factors of charge occur in a given equation. This defect is easily removed by consistent use of 'esu' as the unit of charge, even though it is not fundamental in the same sense as the coulomb is in the MKS system. That consistent use will serve the need of keeping track of how many powers of charge one is dealing with. Once we have introduced the esu, units for electric field, dyne/esu, and for electric potential esu/cm = erg/esu become sensible. The latter two units are defined to be 1 "statvolt" for convenience, in analogy to the volt in the MKS system. (We can then consistently measure E-fields in statvolts/cm.)

On the magnetic side (as in the MKS system) the B-field gets its own unit, the "gauss" rather than $\frac{\text{dyne sec}}{\sqrt{\text{dyne}}\,\text{sec cm}} = \frac{\sqrt{\text{dyne}}}{\text{cm}}$. This serves the same purpose as the introduction of the esu on the electric side - i.e., it enables one to keep track of the number of factors of **B** in a given expression.

Conversions between systems

If the above consistency convensions are followed conversions back and forth can be made via the following simple table which was copied from Purcell.

Quantity	Symbol	Unit, in rationalized MKS System	Equivalent CGS or Gaussian unit
Distance	s	meter	10^2 cm
Force	F	newton	10^5 dynes
Work, energy	W	joule	10^7 ergs
Charge	q	coulomb	2.998 x 10^9 esu
Current	I	ampere	2.998 x esu/sec
Electric Potential	φ	volt	(1/299.8) statvolts
Electric field	E	volts/meter	1/29980) statvolts/cm
Resistance	R	ohm	1.139 x10^{-12} sec/cm
Magnetic field	B	tesla	10^4 gauss
Magnetic flux	Φ	weber	10^8 gauss-cm^2
Auxiliary field	H	amperes/meter	4π X 10^{-3} oersted

If one needs only to convert expressions involving the electric and magnetic field quantities, then the conversion recipe shown on the right is useful[5]. These conversions must be carried out with some care, but going from SI to Gaussian is fairly straightforward. Going the other way is a little more difficult because the μ_o and ε_o do not appear explicitly. For examples see ref. 5.

SI	↔	CGS/Gaussian
μ_o	↔	$1/(\varepsilon_o c^2)$
$4\pi\varepsilon_o$	↔	1
$c\mathbf{B}$	↔	$\mathbf{B}$
$4\pi\mathbf{D}$	↔	$\mathbf{D}$
$4\pi\mathbf{H}/c$	↔	$\mathbf{H}$
$\mathbf{M}/c$	↔	$\mathbf{M}$

[5]Yun-tung Lau, Am J. Phys. **56**, 135 (1988).