Spherical and Cylindrical Coordinate Systems

One coordinate system that we work in is the standard cartesian (x,y,z) system. But, if you are doing a problem with either spherical symmetry (going out the same distance in the x, y, and z directions is the same) or cylindrical symmetry (this means symmetry about one axis), using different coordinate systems may make the problem easier. For example, suppose you are trying to calculate the electric field due to a line of charge lying on the z-axis. The electric field at (1, 0, 0) is the same as the electric field at (0, 1, 0). The only thing that matters is the distance from the z-axis. For this problem, it may be easier to work in cylindrical coordinates.

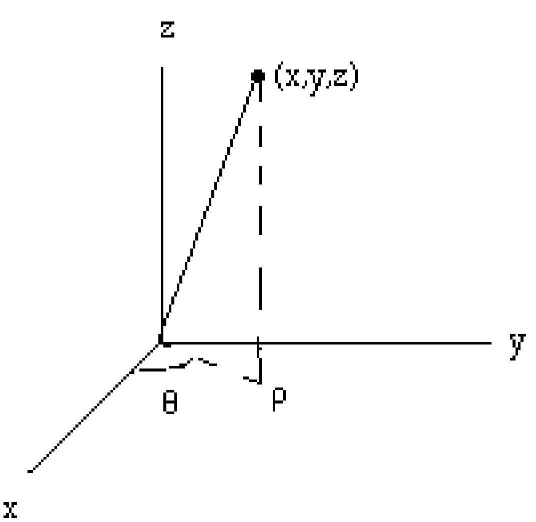
On the other hand, for calculating the field at a point (x, y, z) due to a point charge situated at the origin, it is easier to work in spherical coordinates. If the field is 5 V/m at a point $r = (x^2 + y^2 + z^2)^{1/2} = 10$ cm from the charge, it is 5 V/m at any point 10 cm from the charge.

In cylindrical coordinates, there is an axis of symmetry -- usually chosen to be the z-axis. On any circle parallel to the x-y plane and with center at x=y=0, a cylindrically symmetric function depends only on how far from the z-axis you go. The transformation from cartesian to cylindrical coordinates is, then,

$$x = \rho \cos\theta,$$
$$y = \rho \sin\theta, \qquad (1)$$
$$z = z.$$

Here, ρ is the distance from the point (x,y,z) to the z-axis and θ is the angle between the x-axis and the projection of the vector **r** = (x,y,z) onto the x-y plane. Geometrically, this looks like…

Figure 1



You might also want to do a coordinate transformation from cylindrical coordinates back to Cartesian coordinates. The appropriate coordinate transformation is

$$\rho = \sqrt{x^2 + y^2}$$

$$\theta = \arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right) = \arcsin\left(\frac{y}{\sqrt{x^2+y^2}}\right) \tag{2}$$

$$z = z.$$

Another type of problem is one in which the function depends only on how far radially you go. It is convenient to express the function in terms of r, θ, and φ as defined in fig. 2. The transformation from cartesian coordinates to spherical coordinates is the following:

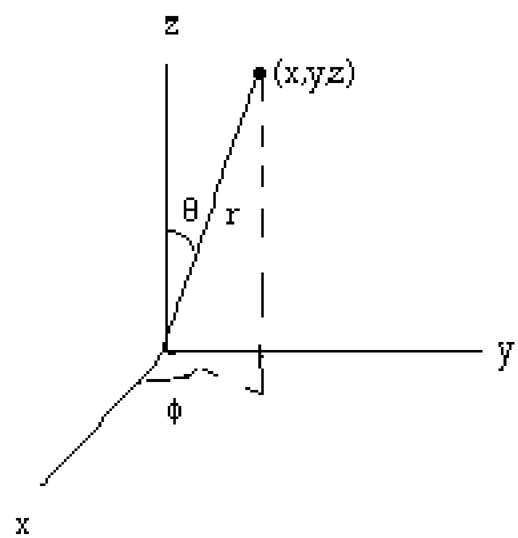
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arctan\left(\frac{\sqrt{x^2 + y^2}}{z}\right) \tag{3}$$

$$\varphi = \arctan\left(\frac{y}{x}\right)$$

The length of the vector **r** = (x,y,z) is r; θ is the angle between the z-axis and **r** and φ is the angle between the x-axis and the projection of **r** onto the x-y plane.

Figure 2



To transform from spherical to Cartesian coordinates, you apply the inverse transformation:

$$x = r\cos\varphi\sin\theta,$$

$$y = r\sin\varphi\sin\theta, \tag{4}$$

$$z = r\cos\theta$$

Some other handy formulae for finding a volume by integration, the gradient and Laplacian of a scalar function u, and the divergence and curl of a vector function **V** in Cartesian, spherical and cylindrical coordinates are given on the following page.