Mathematical Proofs by Induction

How to Do Induction:

We begin by showing a classic example as we explain the formal method. We will show that the sum of numbers from 1 to n is equal to n(n+1)/2. (If you try examples for, say, 1 through 5, you'll even discover the pattern yourself.) In other symbols, [$\sum^{n}_{i=1}$ i] = 1 + 2 + 3 + ... + (n-2) + (n-1) + n = n(n+1)/2.

(1) state the formula for k=1. (we use k so that we can use n for the general case.)	(1) for k=1, [$\sum^{n}_{i=1}$ i] = 1.
(2) assume that the formula works for k ≤ n-1. (see?)	(2) assume that for k ≤ n-1, [$\sum^{n-1}_{i=1}$ i] = (n-1)(n)/2.
(3) state the formula for n in terms of n-1 and fiddle with it until it is an equivalent formula for n.	(3) so, for n, we have [$\sum^{n}_{i=1}$ i] = [$\sum^{n-1}_{i=1}$ i + n] = (n-1)(n)/2 + n = (n-1)(n)/2 + 2n/2 = ((n-1)(n) +2n)/2 = (n^2 - n + 2n)/2 = (n^2 + n)/2 = n(n+1)/2. Q.E.D.

When to Use Induction:

Often one encounters a problem where one must count something, and find a pattern for how to count it. One usually tries several examples, say for one thing through for five things, and guesses at a pattern. Induction is a way of showing that the pattern doesn't change, that it works in all cases.

When Not to Use Induction:

Abstract or theoretical problems almost never can be solved with induction.

Why It Works:

The concept of induction is as follows: we are trying to show that some formula or equation holds for all n. In order to do this, we assume it is true for n-1 and n-2 and so forth all the way down to 1. Then (this is the important part) we use whatever information this gives us to show that it must also be true for n. "Mathematical induction proves that we can climb as high as we like on a ladder, by proving that we can climb onto the bottom rung ... and that from each rung we can climb up to the next one..." (Graham et. al.) As one realizes from this statement, the one thing we left out in the above description is that we have to show that the formula holds for n=1. Knowing that, if we can show that we can construct the formula for n from the one for n-1, then we can always say that n-1 = 1, and so the formula must hold for n=2 ... because it holds for n-1 = 2, the formula holds for n=3, and so on and so forth. That is why induction is so handy for verifying general formulas -- one knows it works for everything, and that it doesn't suddenly stop working after the tenth (or 10^{10}) example.

Reference: Concrete Mathematics by Graham, Knuth, and Patashnik. (also, mentally, notes from Marlin DeWeerdt.)