

Chapter 5

Quasi-static Internal Dynamics

In this chapter, we focus on the dynamics of individual grains, based on precision measurements of the internal velocity field, and time-resolved trajectories of particles during both the steady states and transient responses. The dynamics is investigated in the quasi-static regime, in which particles are nearly in static equilibrium and maintain contacts with multiple neighbors. Theoretical justifications of the range of normal stress and driving rates for generating quasi-static flows are provided in Appendix A.1. In Section 5.1.1, the contrasting behavior of the disordered and crystallized states of the flows is analyzed in detail. In Section 5.1.2, we examine the roles of the particle size and the system size (the thickness of the packing) on the mean velocity fields. The rate-independence of the velocity field and the proportionality of the shear force to the normal load are described in Section 5.1.3 and 5.1.4. In Section 5.2, we analyze the internal motion captured during the reversal of shearing; investigations of this transient behavior provide insights into the rheology of a quasi-static granular bulk beyond the steady state. The effects of stationary boundaries on shear banding, comparisons of quasi-static granular flows in different geometries, the gradual development of bulk properties of a granular packing as its thickness increases, and the contrast between a sheared granular packing to ordinary fluids are discussed in Section 5.3; the discussions conclude with a heuristic model as an attempt to explain the observed shear banding.

Throughout this dissertation, we use the following conventions for the three mutually orthogonal coordinates: x refers to the horizontal direction of the mean flow in the region of imaging; the z axis points vertically upwards with $z = 0$ marking the height of the interface between the rotating boundary and the beads; the y axis is perpendicular to both x and z and points towards the center of the annular channel, with $y = 0$ denoting the location of the outer wall.

5.1 Steady flows

5.1.1 Effects of crystalline order

Dramatic differences in both the ordering and the shape of the velocity field are found before and after the crystallization transition. Fig. 5.1 illustrates the three-dimensional features of these changes. In the disordered state shown in (a), the grain velocity exhibits a significant

decay towards the sidewalls. The ordered state (b) forms hexagonal sheets sliding against adjacent layers, and shows no significant decay of speed towards the sidewalls. Point defects and dislocations are visible from the sample image in case (b); the time-resolved image sequence (with movies available via the online archive associated with Ref. [56]) show that point defects often persist as they move across the field of view.

Fig. 5.2 shows the time-resolved trajectories of particles sampled at horizontal internal planes. In each graph, coordinates of centers of all particles captured in a horizontal (xy) region of size $8d$ by $8d$ with a depth of field of about $1d$ (set by the thickness of the laser light sheet) are overlaid as functions of time. In a disordered state shown as graph (a), the grains show no obvious spatial correlations, and the trajectories here typically end within a net travel distance along y comparable to the grain diameter, presumably as a consequence of particles' leaving the depth of field. In contrast, Fig. 5.2(b) shows that grain motions in the crystallized state are strongly correlated across the entire field of view. Fig. 5.2(c) illustrates longitudinal motions of crystallized grains, showing the collective translation that is temporally non-uniform. The behavior of crystallized layers is consistent with the zig-zagging of hexagonal structures over adjacent layers, as shown in online movies associated with Ref. [56]. Similar zig-zag patterns have also been seen in experiments and simulations on other physical systems, such as suspended colloids [49, 41]. In the quasi-static flow investigated here, the typical patterns only depend on the state of order but not on the actual shear rate: for a given state of order, the behavior is similar regardless of the large differences in local time scales at different heights and with different driving speed. For instance, the characteristic periods of the zig-zagging in Fig. 5.2(b,c), shown by horizontal bars on the graph, consistently correspond to a few particle diameters of local translation, or to the local shear strains of the order of unity, because these patterns are a consequence of the relative motion of stacked hexagonal planar structures.

The measurement of the vertical profiles of mean velocity for different final states is presented in Fig. 5.3. For either the crystallized or the disordered state, the sampling starts after the packing has evolved under unidirectional shearing at a speed of $12d/s$ for several days, corresponding to 10^6d of total translation of the upper boundary. This ensures that the velocity field has reached the asymptotic state. Since the height-dependent grain velocity varies over several decades, we extract particle velocities from local imaging of

patches recorded at different frame rates (as illustrated in Appendix B); each data point is determined from a local average over $10^2 d$ of accumulated particle displacement. The amount of stochastic variation of the mean value under this finite-length sampling is of the order of 30%, labeled as δ on the graph. Further information regarding measurement uncertainty is provided in Appendix B. For the disordered final state created with the same amount of glass beads, the height-dependent asymptotic velocity is measured at two vertical planes during one experiment. This is because, unlike the crystallized state, the grain velocity of the disordered state decays towards the sidewalls (Fig. 5.1). But the decay is generally not more than a factor of 10; there exists a finite slip velocity at the sidewalls.

Our measurements show that the downward decay of the asymptotic mean velocity in the crystallized state is significantly steeper than in the disordered state. The change of velocity profile due to crystalline order is also qualitatively reflected in Fig.3.1(d), which shows a step decreases of particle velocity near the bottom at the ordering transition. A plausible explanation is that the stronger decay of velocity in the crystalline state is a consequence of the reduction of downward transfer of horizontal momentum due to the formation of hexagonally ordered horizontal sheets (Fig. 4 of Ref. [56]) which slip over adjacent layers in a coherent way. This behavior is consistent with the measured reduction of shear force. Also included in Fig. 5.3 is the velocity profile of a thicker packing (in this case 36 layers), which reaches a stationary state with its degree of local crystalline order gradually varying with height (more ordered on the lower part); in this case, the velocity profile of this non-uniform state lies between the uniformly ordered and disordered states.

The smooth sidewalls can induce crystalline order that is localized to their vicinity and oriented quite differently than the horizontal planes of the globally crystallized state, as is revealed by the internal imaging at different planes. For example, in the images for calculating the disordered-state velocity profiles in Fig. 5.3, this hexagonal structure is visible at one vertical plane that is closer to the sidewall than the other plane, in which grain configuration appears to be random. In Fig. 5.3, we choose the bin width for the disordered state in such a way that each of the triangles corresponds to one row in the 22-layer side-wall induced structure. Nevertheless, the presence of this sidewall induced local order does not have a strong impact on the velocity profile, because the sidewall induced structure is not oriented in a way that favors slip between layers oriented perpendicularly

to the imposed velocity gradient. This is a useful example showing that the local rheology of granular flows is highly anisotropic.

5.1.2 Effects of system size and particle size

In this subsection, we investigate the development of the non-linear velocity profile with increasing packing thickness, and the effects of particle size. We limit our examples here to uniformly ordered states to maintain comparability. The fact that the crystallized packing flows like stacked semi-rigid horizontal sheets (Sec. 5.1.1) with a vertical profile of velocity insensitive to the choice of measurement plane provides an additional convenience in making comparisons.

We present in Fig. 5.4 the time-averaged velocity profiles measured on an internal vertical plane (xz), using different amounts of glass beads which form crystallized states of 5, 8, 12, 18, and 24 layers, respectively. Each data point represents the mean velocity and the height of a well-defined layer.

Note that in a shear flow as thin as 5 layers of material, the velocity profile is almost linear, approaching zero at the bottom and its maximum value at the top; in this case the shear rate is uniform. However, as the thickness increases, the normalized velocity profile progressively deviates from the linear relation. (This deviation can be seen on Fig. 5.4, by comparing the data points with the corresponding hypothetical linear relation $V(z) = 1 - |z/H_0|$.)

The velocity profiles of flows in these crystallized states gradually approach a master curve that is insensitive to the exact location of the bottom boundary, as thickness is increased. Here the particle diameter d is used to non-dimensionalize the vertical coordinate. The master curve is approximately quadratic on a semi-log plot, as shown on the graph; the fitted velocity profile is equivalent to a Gaussian curve centered at $z \approx +2.2d$. The dominant local length scale of the velocity decay with height can be extracted by defining a spatial decay rate $\lambda(z) = (dV(z)/dz)/V(z)$ and substituting the Gaussian fit for $V(z)$; the result gives $\lambda(z) = (0.103 - 0.02375(2z/d)) \cdot d^{-1}$. (For instance, $\lambda^{-1} \approx 5.1d$ at $z = -2d$; $\lambda^{-1} \approx 2.9d$ at $z = -5d$ etc.) We have also verified and show in Fig. 5.3 that, as long as a specific state of order is created, the roughness of the bottom (ranging from a flat

surface to an irregular surface with a grain-scale bumpiness) does not significantly affect the internal velocity field except for regions within about $5d$ from the bottom. In the 24-layer case, the velocity field of nearly the entire packing is more sensitive to the state of internal order than to the roughness of the bottom boundary. In general, as long as the packing is sufficiently thick, the bottom boundary affects the steady-state velocity field of a sheared packing only by its influence on the routes of evolution that lead to different states of internal order, as described in Chapter 4.

We also measure the time-averaged velocity fields using different sizes of mono-disperse spherical glass beads, as shown in Fig. 5.5. These experiments are performed with a flat bottom boundary: the data points here include flows of three different particle sizes at a comparable layer number (14 or 15), plus a 24-layer uniformly crystallized flow. (The 15-layer flow for $d = 0.68\text{mm}$ is only partially disordered on a flat bottom, as is described in Section 3.3.) Notably, the velocity decay of different particles follows roughly the same trend in terms of physical distance from the driving surface. Even though particle diameter d may seem a convenient length to non-dimensionalize coordinates, the inset shows a clear dispersion among velocity profiles for the same state of order but of different particle sizes, when the coordinate is scaled as z/d . For both crystallized or disordered states, and different particle sizes, the vertical decay of particle velocity is generally steeper than that of ordinary fluids in the same channel; the curves for the hypothetical fluid flows have a primary decay length $\lambda_1^{-1} = (W_0/\pi)$ set by the channel width W_0 , as derived in Appendix A.2. The implications of these velocity decays are discussed further in Section 5.3.

An additional feature of the velocity profiles for these experiments using a flat bottom is the common trend shared by all data: a slightly up-rising tail occurs near the flat bottom; see Fig. 5.5. The extreme case is the velocity profile for $d = 2.0\text{mm}$, which become flat for the lowest 6 layers. In addition, the comparison of the three velocity profiles of 24-layer crystallized states using different bottom conditions (shown as Fig. 5.3) suggests that the rising tail is unique to the flat bottom condition. The underlying mechanism for this behavior is unclear. Furthermore, inspecting the image sequences and time-resolved trajectories for the 2mm-particle data reveals that the lowest six layers translate as a rigid block. Particles right above the ‘interface’ translate in a stick-slip fashion relative to the rigid block below; the typical steps of the stick-slips are rather large (about 10^1d).

Interestingly, despite of the dramatic stick-slips between the top of this rigid block and the region above, the vertical profile of time-averaged velocity does not show a discontinuity, except for an abrupt change of the slope on this semi-log plot.

5.1.3 Invariance for changes in driving speeds and normal loads

In Fig. 5.6, we show that the asymptotic mean velocity of grains scales almost perfectly with driving speed over a range of two decades of driving speeds in our experiments, for flows of the same thickness and state of ordering. This behavior is consistent with the notion that the packing is driven in a quasi-static state, for which the coarse-grained local displacement at a given depth is simply proportional to the accumulated displacement of the driving boundary. In Section 5.2.1, we also show that in the speed range of our experiments, the temporal duration of the transient response following reversal of the driving speed also has a simple (inverse) scaling with driving speed.

Our smallest normal load of $1.3\text{Kg}\cdot g$ is much larger than the total weight of the grains in all cases. In the case of a 24-layer packing, this normal load is more than ten times the net weight of the grains immersed in the fluid. We also vary the normal load by about a factor of 4 and measure the velocity profiles, shown as Fig. 5.7; the invariance of the velocity profiles suggests that the normal load is sufficiently large that gravity plays an insignificant role in its dynamics. However, our largest normal load is much smaller than the load required to significantly deform or damage the glass beads, which could result in stochastic internal shear bands as reported in simulations [1] and experiments modeling geological systems. The change of overall thickness caused by varying the normal load is less than 1 percent.

5.1.4 Effective macroscopic friction

Within the range of our experimental parameters, the boundary shear force is almost proportional to the applied normal force, and does not show detectable change with respect to change of driving speeds over two decades. The effective macroscopic friction coefficient, defined as the ratio of shear stress to normal stress, is about 0.31 for the disordered

state and 0.26 for the crystallized state -see Fig. 3.1 for the change at the crystallization transition. These values are close to the typical values reported in the simulation of Ref. [1], but significantly smaller than the friction coefficients shown simulations reported in Ref. [22] and Ref.[59]. The discrepancy may be partly due to different settings of material parameters. Also, by comparing the measured shear force between dry-particle and immersed-particle experiments, we find that the presence of the immersion fluid reduces the macroscopic shear force by about 12 percent in our experiments, presumably due to lubrication.

5.2 Transient response to shear reversals

To obtain deeper insight into the quasi-static grain dynamics, we extend our investigation from steady states to transient responses. Here we focus on a particular type of transient — the brief period during which a sheared packing readjusts its internal structure to a new steady state after the direction of shearing has been reversed. During this period, grains appear anomalously mobile compared to their steady-state behavior. We refer to this behavior as a shear-reversal transient, which may be regarded as a brief period that the material is “soft”. The investigation into the transient behaviors of dense granular flows may provide useful information for going beyond a mean-field description.

5.2.1 Grain motions during the shear-reversal transient of a thick packing

For either the disordered or the crystallized state, one can stop the steady shearing and then resume the boundary motion in the opposite direction. The reversal of shear causes extraordinary motions of internal grains, and successive sinking of the driving boundary under a fixed normal load. In this section, we use a typical packing thickness (200g of glass beads of diameter $d = 0.68\text{mm}$, which would form a 24-layer structure when crystallized) as an example to illustrate the generic behavior of the shear-reversal transient.

Fig. 5.8 demonstrates the displacements of individual grains sampled on an internal vertical (xz) plane of a disordered state, whose boundary motion is being switched from $+x$ to $-x$. During the first $12d$ of travel of the boundary motion in the $-x$ direction (b),