

Math 222 Fall 2002 Midterm Test

Due: Fri. Nov. 8, 5 PM

to the inbox outside my office in Koshland H207

(or hand in during classtime by Nov. 8)

This is a open-book test. You may use your notes, homework and lab assignments, textbook, and my solutions. You must work alone on this test.

There is no time limit on completing the test. **You will need either a calculator or Mathematica.** Indicate on your test where you used the calculator or Mathematica. If you use Mathematica, please provide a printout of your Mathematica notebook. When you are done, please sign the Honor Code pledge below.

I accept full responsibility under the Haverford Honor System for my conduct on this test:

Sign your name (after completing test) _____

Problems:

1. (a) Work out in detail two steps of Newton's Method **in the LUCKY-7 number system** for solving the equation $\sin x = 0$ starting at the point $x_0 = 0.1$, assuming that for each operation you compute, the computer first determines the answer to arbitrary precision and then rounds the result to the nearest LUCKY-7 number before going on to the next computation. Make sure to show the results of all of these intermediate computations. What type of convergence do you see to the true solution $x = 0$?

(b) Repeat part (a) for the equation $1 - \cos x = 0$ (once again, with initial point $x_0 = 0.1$ and true solution $x = 0$).

(c) Why is there a qualitative difference in the convergence rates for parts (a) and (b)?

(d) In which of the 4 computations (x_1 for \sin , x_2 for \sin , x_1 for $1 - \cos$, and x_2 for $1 - \cos$) did roundoff effects cause the largest relative error? Why?

2. (a) Work out one step of the steepest descent algorithm for finding a local minimum of $f(x, y) = x^4 + 2y^4 - 2x + 2y + 4xy$ starting at $(x, y) = (0, 0)$.

(b) Work out one step of the application of Newton's method to finding a local minimum of $f(x, y) = x^4 + 2y^4 - 2x + 2y + 4xy$ starting at $(x, y) = (0, 0)$.

3. On HW # 4 Problem 4, you looked at the accuracy of the “midpoint-rectangle area” A_{mid} as an approximation to the area A under the curve $y = f(x)$ from $x = c - \frac{b-a}{2N}$ to $x = c + \frac{b-a}{2N}$.

Now consider another approximation: draw the diagonal line from $(c - \frac{b-a}{2N}, f(c - \frac{b-a}{2N}))$ to $(c + \frac{b-a}{2N}, f(c + \frac{b-a}{2N}))$ and compute the area A_{diag} under this diagonal line as your approximation to A (see picture below).

(a) What is the formula for A_{diag} in terms of f , a , b , and c ? (Hint: the region is a rectangle plus a triangle).

(b) Use the Taylor series expansion of $f(x)$ about $x = c$ to determine a series expansion for A_{diag} .

(c) Determine the order of the error in approximating A by A_{diag} as a power of $1/N$, i.e., is it $O(1/N)$ or $O(1/N^2)$ or $O(1/N^3)$, etc.?

(d) Based on these computations and those on HW # 4 Problem 4, do you expect A_{diag} to be a better or worse approximation to A than A_{mid} ? Why?

