Math 222 Fall 2002—Lab #4 Due: Mon. Nov. 18

Reading (from Cheney and Kincaid): Ch. 4.1 pp. 136–139, 4.2 pp. 162–165, 4.3 pp. 172–174, 7.1 pp. 318–324, 7.2 pp. 330–35, pp. 342–344, 10.1 pp. 438–443, 10.2 pp. 446-449, 10.3 p. 462.

Assignment: Do the "Spline" problem and one of the two "Least Squares" choices. For each of the Least Squares problems, I've posted a skeleton notebook on the Web page that outlines what you need to do and includes the data (so you don't have to type it in).

Spline Problem: Depth of the water table

Given any three distinct points (x_1, y_1), (x_2, y_2), and (x_3, y_3) in the plane, and values of some function at each of these points (call these values z_1, z_2, and z_3), work out how you would find a linear function $f(x, y) = Ax+By+C$ such that $f(x_1, y_1) = z_1$, $f(x_2, y_2) = z_2$, and $f(x_3, y_3) = z_3$. Write a Mathematica function `patch[pt1,pt2,pt3,x,y]` which outputs $f(x, y)$, given three 3-vectors $pt1 = \{x1, y1, z1\}$, $pt2 = \{x2, y2, z2\}$, and $pt3 = \{x3, y3, z3\}$, and (x, y) lying inside the triangle with vertices $(x1, y1)$, $(x2, y2)$, $(x3, y3)$.

Wells are drilled at 9 positions (i, j) for $0 \le i, j \le 2$, and the depth of the water table below the earth's surface is recorded at each point, as shown below:

Position (km)	Depth of water table (m)
$(0, 0)$	6
$(0, 1)$	12
$(0, 2)$	8
$(1, 0)$	8
$(1, 1)$	11
$(1, 2)$	12
$(2, 0)$	6
$(2, 1)$	10
$(2, 2)$	8

Partition the square $0 \le x, y \le 2$ into 8 triangles, using the lines $x = 0$, $x = 1$, $x = 2$, $y = 0$, $y = 1$, $y = 2$, $x + y = 1$, $x + y = 2$, and $x + y = 3$. Now construct a piecewise linear function $g(x, y)$ that matches the well measurements above and is continuous (all you need to do is work out which triangle you're in and call `patch` with the appropriate arguments). Use the command `Plot3D` to graph g:

```
Plot3D[ g[x,y],{x,0,2},{y,0,2},PlotPoints->30]
```

(If your surface looks continuous, without any "cliffs", you've probably done it correctly.) What is the approximate depth of the water table at $(0.4, 0.2)$? At $(1.1, 0.9)$?

Least Squares Choice 1: Dow Jones

The list below gives the levels of the Dow Jones Industrials average at the end of each year from 1976 through 2001.

1005, 831, 805, 838, 963, 875, 1066, 1258, 1211, 1546, 1895, 1938, 2168, 2753, 2633, 3168, 3301, 3754, 3834, 5117, 6448, 7908, 9181, 11497, 10786, 10021

(a) Find the function in the family $y = be^{ct}$ that is the best fit to this data in the least-squares sense. What does this model predict for the level of the Dow in 2020?

(b) Another approach to this same problem is to observe that if $y = be^{ct}$, then $\ln y = \ln b + ct$, so that if you take the ln of the Dow levels, you can look for the linear function $M + Nt$ that is the best fit to this data in the least-squares sense. Do this. What does this model predict for the level of the Dow in 2020?

(c) Compare your results from parts (a) and (b). What is the maximum difference in Dow level between these two models for $1976 \leq t \leq 2001$.

Least Squares Choice 2: Drug decay

Trimetrexate is an AIDS drug that is administered regularly into the bloodstream; its level in the bloodstream decays as it is absorbed into the rest of the body. Below is a set of measurements of trimetrexate levels in the bloodstream (in mg/m^2) every hour from $t = 0$ hours to $t = 30$ hours:

94.8, 58.0, 39.1, 26.9, 20.2, 18.7, 14.3, 14.2, 12.7, 10.0, 8.1, 7.8, 6.6, 6.8, 5.6, 4.4, 4.6, 3.8, 4.5, 3.7, 3.4, 2.4, 2.7, 2.1, 1.7, 0.8, 0.8, 1.0, 1.0, 0.4, 1.2

(a) As a first model, assume that the drug level y depends on time according to $y = be^{ct}$. This means that $\ln y = \ln b + ct$, so that if you take the ln of the drug levels, you can look for the linear function $M + Nt$ that is the best fit to this data in the least-squares sense. Do this. What does this model predict for the half-life of trimetrexate (the time it takes for an initial amount to decay to half that amount)?

(b) A more sophisticated model assumes the "biexponential form" $y = b_1 e^{c_1 t} + b_2 e^{c_2 t}$, assuming, for example, that some organs in the body absorb the drug more quickly than others. Perform a nonlinear least squares fit of the data to this form. You should get a significantly better fit.