Math 222 Fall 2002—Lab #3 Due: Fri. Nov. 1

Reading (from Cheney and Kincaid): material from Ch. 11 as on HW # 4

Assignment: Do one of the two "Simulation" choices and one of the two "Metropolis Monte Carlo" choices. See `www.haverford.edu/math/rmanning/math222/hw.html` for notebook with some useful commands.

Simulation Choice 1: Blackjack

You have developed card-counting skills that allow you to win 51% of the time in blackjack. In other words, if you bet \$ b, you have a 51% chance of winning an addition \$ b (in addition to your bet), and a 49 % chance of losing your \$ b. Ignore fancy things like "doubling down" and "splitting"; assume I have incorporated it all into the 51 % figure.

Your goal is to take the \$5000 in your bank account, go to Atlantic City, and turn that \$5000 into \$20000. Write a function in Mathematica `gamble[b_]` that will simulate your gambling fate; the function `gamble` should return 1 if you reach \$20000 before you reach \$0 (I'll call this "success"), return -1 if you reach \$0 before you reach \$20000 (I'll call this "failure"), and return 0 if you reach neither in the first 5000 games.

Run `gamble` 100 times with $b = 5000$ and tally the number of successes and failures; you may want to try this a few times to get a sense of how stable your results are. How do your prospects change if you lower b to 1000? Why does that make sense? Over approximately what range of b do you succeed at least half of the time?

Simulation Choice 2: Chemical Spill

Oh no! The ACME Chemical company has spilled 4000 liters of benzene at location $(0, 0)$ in the state of Flatland. It has seeped into the ground, and is beginning to spread. The residents of the city of Peril (located in the square region $5 \leq x \leq 7, 5 \leq y \leq 7$, all positions in miles) are worried for their health. They know that if the total level of benzene reaches 30 liters underneath their city, their water will be unhealthy. The Perilites have asked you to model the spread of the benzene over the next year to see if they will be safe.

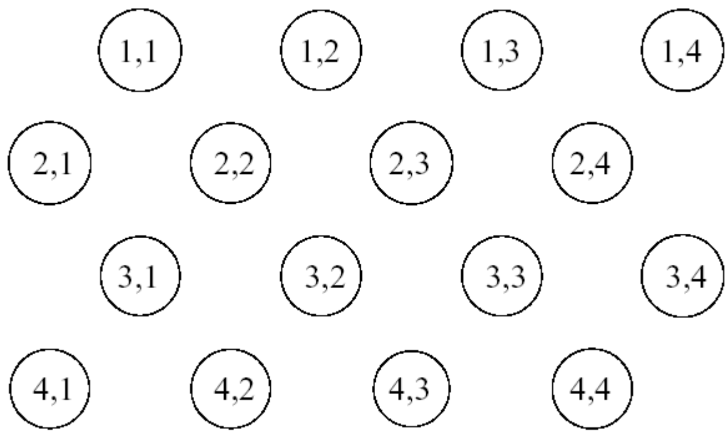
Model the benzene as 4000 separate 1-liter "particles", all sitting at $(0, 0)$ at time $t = 0$. In each day, assume that each particle has a 25% chance of moving 1 mile north a 25% chance of moving 1 mile south, a 25% chance of moving 1 mile east, and a 25% chance of moving 1 mile west.

(a) Run a simulation of the 4000 benzene-liters for 365 days, keeping track of how much sits under Peril each day. Plot the benzene level under Peril versus time. What is the peak level of benzene? What is the qualitative shape of the curve? Why does that make sense? Be sure to repeat the simulation a few times to be sure you have stable results.

(b) The Perilites have erected a pumping facility on the rectangle $3 \leq x \leq 6, 3 \leq y \leq 4$. This facility can remove up to P liters of benzene per day from the ground beneath the facility (of course, on days when less than P liters lies in the rectangle, it only removes what is there, not a full P liters). Run a simulation to determine how high the Perilites should set the pumping capacity P to be reasonably sure that the benzene level within Peril will stay below 30 liters for the whole year.

Metropolis Monte Carlo Choice 1: Ising Model

Consider an L-by-L "triangular lattice" ($L = 4$ shown below): Each central cell has six "nearest



neighbors", all the same distance away. If i is odd, cell (i, j) has nearest neighbors $(i-1, j)$, $(i-1, j+1)$, $(i, j-1)$, $(i, j+1)$, $(i+1, j)$, and $(i+1, j+1)$. If i is even, cell (i, j) has nearest neighbors $(i-1, j)$, $(i-1, j-1)$, $(i, j-1)$, $(i, j+1)$, $(i+1, j)$, and $(i+1, j-1)$. Assume "periodic boundary conditions": if you walk off the top, you re-enter at the bottom; if you walk off the left, you re-enter at the right, etc. Then, *every* cell has six equidistant nearest neighbors (use rules given above, but if $i-1$ or $j-1$ is 0, change it to L; if $i+1$ or $j+1$ is $L+1$, change it to 1).

The Ising model is often used to study phase transitions, magnetism, etc. At each lattice point (i, j), we place a number s_{ij} that is either 1 ("spin up") or -1 ("spin down"). To a given choice of the s_{ij} values, we associate an energy:

$$E = -J \sum_{n.n.(i,j),(k,l)} s_{ij} s_{kl}$$

where the sum is taken over all pairs of nearest neighbors (i, j) **and** (k, l).

Consider first $L = 6$ and $J = 1$ (a "ferromagnetic Ising model"). Choose a random initial array of s_{ij} values (each -1 or 1), and run Metropolis Monte Carlo to minimize E. What solution do you converge to? Does that make sense as the global minimum?

Consider next $L = 6$ and $J = -1$ (an "antiferromagnetic Ising model") Choose a random initial array of s_{ij} values (each -1 or 1), and run Metropolis Monte Carlo to minimize E. Show the time-series of E over your run. What is the minimal energy? Use the `view` function I wrote in the Lab 3 sample notebook on the course Web page to display the minimizing state. Why does it make sense that the solution is more complicated now? Try this case a couple of times to see if you get the same results.

Repeat for a $L = 16$ antiferromagnetic model. (**Note: Runs will take a long time, maybe 15 minutes or more. Be careful not to take too many steps, but enough to get**

convergence). Do you see any patterns? Try a few runs; do you get the same minimum energy or do you land in different local minima?

Metropolis Monte Carlo Choice 2: Traveling Salesperson

Given a set of N cities at positions $(x_1, y_1), \cdots (x_N, y_N)$ in $\mathbb{R}^2$, we seek an order in which to visit the cities so as to minimize the total distance traveled. So, you can start at any city, and must then go to each of the other $N-1$ cities exactly once.

Consider first $N = 9$ cities lying on an ellipse $(x_j, y_j) = (\cos(2\pi(j-1)/9), a\sin(2\pi(j-1)/9))$, $0 < a < 1$. Run Metropolis Monte Carlo to solve this problem for $a = 0.9$. Show the time-series of E over your run. What is the minimal total distance? Use the `routeview` function I wrote in the Lab 3 sample notebook on the course HW Web page to display the minimizing state. Does that makes sense as the global minimum?

Repeat for a value of a small enough that a different route is the global minimum. Try a few runs. Do you get the same optimal route or do you land in different local minima?

By experimenting, determine approximately the value of a where the original route ceases to be optimal.