Math 222 Fall 2002—HW #3 Due: Tue. Oct. 8

Reading (from Cheney and Kincaid):

Chapter 14 introduction

§14.1 but not Fibonacci section or later references to Fibonacci search

§14.2 but not Nelder-Meade or Simulated Annealing sections

Problems (careful: don't do the "Computer Problems"):

1. Consider the following three methods for locating a local minimum of a function $f(x)$ between $x = 0$ and $x = 1$ (we discussed each in class).

(a) Golden Section search

(b) Lee I: At step n, we have $0 \leq a_n < c_n < b_n \leq 1$, with c_n the midpoint of $[a_n, b_n]$, and $f(c_n) < f(a_n), f(b_n)$, so we know there is a local min between a_n and b_n. Now compute f at d_n and e_n, the midpoints of $[a_n, c_n]$ and $[c_n, b_n]$. For one of the following three intervals, f will be lower at the midpoint than at the endpoints: $[a_n, c_n]$, $[d_n, e_n]$, $[c_n, b_n]$. Keep this interval for the next iteration, calling its left endpoint a_{n+1}, its right endpoint b_{n+1} and its midpoint c_{n+1}.

(c) Lee II: Do the same thing as Lee I, but instead of computing $f(d_n)$ and $f(e_n)$ each time, first compute $f(d_n)$. If it is lower than $f(a_n)$ and $f(c_n)$, then accept $[a_n, c_n]$ as your new interval and go on to the next iteration. If not, compute $f(e_n)$, and accept either $[d_n, e_n]$ or $[c_n, b_n]$ as the new interval, depending on which has f at the midpoint lower than f at the endpoints.

That's a complicated setup, but my question is simple: At how many different points will we need to compute f in order for each algorithm to compute the local minimum to within 10^{-6}, assuming we start with $a_0 = 0$, $b_0 = 1$, and $c_0 = 1/2$? For (c), assume that, on average, 1/3 of the time $f(d_n)$ is lower than $f(a_n)$ and $f(c_n)$ (using the logic that of the 3 candidate intervals from Lee I, each is equally likely to be "the one"). So, the "average" number of f computations per step is $(1/3)(1) + (2/3)(2) = 5/3$.

2. Say we are looking for a local minimum of $f(x) = -x^2 + x^4 + 2$ by using Newton's Method. For which initial guesses will the iteration converge to the local *maximum* of f? (Hint: use the same sort of geometric argument you used in Problem # 5 of HW # 2).

3. Say we are looking for the minimum of $f(x, y) = x^2 + 4y^2$. For a current guess (x_n, y_n), work out on paper what the next guess (x_{n+1}, y_{n+1}) will be following the steepest descent algorithm (for this simple function, you can work out the whole process and get a formula for x_{n+1} and y_{n+1} in terms of x_n and y_n, though there is a fair bit of calculation required to get there).

Using this result, starting at $(x_0, y_0) = (2, 3)$, find the first 5 steps of the steepest descent algorithm. Does the convergence appear to be linear? quadratic? How does the convergence compare to Newton's Method for this problem?