Math 222 Fall 2002—HW #1 Due: Fri. Sep. 13

Reading (from Cheney and Kincaid):

Chapter 1, pp. 4–5 (Sections "Errors: Absolute and Relative" and "Rounding and Chopping")

§1.2, pp. 22–31. (Ignore the insult in the first sentence. Throughout the semester, you may ignore sections of the text about Maple, Mathematica, Matlab, etc.)

§2.1, pp. 44–46 §2.2, pp. 54–61, 63–65 §2.3, pp. 74–77

Problems (careful: don't do the "Computer Problems"):

1. §1.2, # 1. (Show that for $f(x) = (1+x)^n$, Eq. (6) takes the form

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

For $n = 2, 3, 1/2$, give the particular form of this series. Use the last form to compute $\sqrt{1.0001}$ correct to 15 decimal places (rounded).)

2. Example 5 shows that $\ln(1+x) = \sum_{k=1}^{\infty}(-1)^{k-1}\frac{x^k}{k}$ for $-1 < x \leq 1$. How many terms of this series would you need to compute $\ln 2$ accurate to within 10^{-5}? How many terms would be required to get the same accuracy if you first used the analytic trick $\ln 2 = \ln\left(e\frac{2}{e}\right) = 1 + \ln\left(\frac{2}{e}\right)$? (Moral: "A bit of thought before computing can make a big difference")

3. Recall the base-10 `LUCKY-7` system from class: Numbers get stored as $a_1.a_2 \cdots a_7 \times 10^c$, where $a_j = 0, 1, 2, \ldots$ or 9, $a_1 \neq 0$ and $-40 < c < 40$. Find numbers a, b, and c so that, in computations on a `LUCKY-7` computer, $a + (b + c) \neq (a + b) + c$.

4. Later in the course we will see that $f''(a)$ is often approximated as follows:

$$f''(a) \approx \frac{f(a+h) - 2f(a) + f(a-h)}{h^2},$$

for some small h. Using the Taylor series expansions of $f(a+h)$ and $f(a-h)$ about a, verify that the leading-order term of this expression is, in fact, $f''(a)$, and determine the order of the approximation (i.e., is it correct to $O(h)$, $O(h^2)$, etc.)? You will need to experiment to determine how many terms in the Taylor expansions are necessary to find both the leading-order term and the first additional non-zero term (which gives you the accuracy of the approximation).

5. Consider implementing the approximation from Problem 4 on a LUCKY-7 computer for the function $f(x) = \sqrt{x}$ at $a = 1$. Imagine that the computer can determine $f(x)$ exactly for any x, but then must store it as the nearest LUCKY-7 number before plugging it into the quotient used to approximate $f''(1)$.

Compute the exact value of $f''(1)$. Next, consider a range of h values between 10^{-3} and 1 and compute the quotient *in the LUCKY-7 number system*. How does the accuracy of the approximation depend on h in this range? Why does it depend on h in this way, in terms of the types of errors we discussed in class? Estimate the value of h in this range that gives the best approximation to $f''(1)$.