

Math 116 Fall 2003—Solutions to Practice Problems for Midterm

1. Consider the following set of data:

12.1 11.7 1.7 10.6 14.6 14.4 -5.2 -1.3 14.6 15.3
10.4 13.2 15.9 13.4 18.5 11.7 13.9 14.9 5.1 10.6
1.3 15.4 9.3 12.0 -0.2 5.7 -2.1 13.5 10.2 12.8

(a) Construct a stem-leaf plot or a histogram of the data and comment on key features of the data.

Here's a stem-leaf plot, using 3 "bins" per tens digit (L for 0.0 thru 3.3, M for 3.4 thru 6.7, and H for 6.7 thru 9.9):

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1H  8.5
1M  4.6 4.4 4.6 5.3 5.9 3.4 3.9 4.9 5.4 3.5
1L  2.1 1.7 0.6 0.4 3.2 1.7 0.6 2.0 0.2 2.8
0H  9.3
0M  5.1 5.7
0L  1.7
-0L 1.3 1.3 0.2 2.1
-0M 5.2
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There is a strong primary peak in the low-to-mid 10's, and there also appears to be a second smaller peak just below 0. The primary peak appears quite symmetric; there is too little data to tell about the symmetry about the secondary peak.

(b) Compute the median, mean, and standard deviation of the data

There are 30 data points; the 15th and 16th largest are 12.0 and 11.7 respectively, so the median is $(12.0 + 11.7)/2 = 11.85$.

The mean is:

$$\bar{x} = \frac{\sum_{i=1}^{30} x_i}{30} = \frac{294}{30} = 9.8.$$

The variance is:

$$s^2 = \frac{\sum_{i=1}^{30} x_i^2 - 30(\bar{x})^2}{29} = \frac{3998.96 - 30(9.8)^2}{29} = 38.54344,$$

so that the standard deviation is:

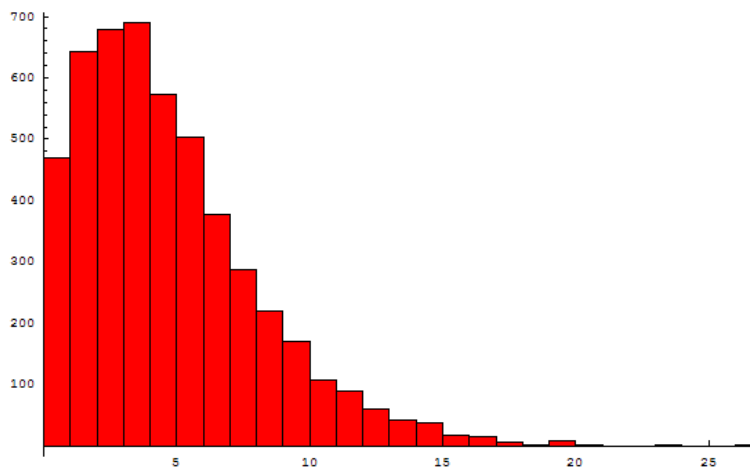
$$s = \sqrt{38.54344} = 6.208.$$

(c) Construct a box-plot of the data.

We know that the median is 11.85. The top half of the data contains 15 data points, so its median is the 8th largest point, which is 14.4. The bottom half of the data contains 15 data points, so its median is the 8th smallest point, which is 5.7.

So, the boxplot has a line from the smallest point -5.2 to the largest 18.5 , and within that, a box extending from 5.7 to 14.4 , with the median of 11.85 marked in the middle.

2. Consider the following histogram for a set of data:



Which one of the following is the most plausible set of values for the median, mean \bar{x} and standard deviation s of this data? Explain.

- (a) median = 4.7, \bar{x} = 4.0, s = 1.4, (b) median = 4.7, \bar{x} = 4.0, s = 3.4
(c) median = 4.7, \bar{x} = 4.0, s = 10.4, (d) median = 4.0, \bar{x} = 4.7, s = 1.4
(e) median = 4.0, \bar{x} = 4.7, s = 3.4, (f) median = 4.0, \bar{x} = 4.7, s = 10.4

The data is skewed to the right, so the median should be smaller than the mean, thus ruling out a, b, or c. In addition, $\bar{x} \pm s$ should contain a sizable chunk, but not the vast majority of, the data, so $s = 3.4$ is the most plausible value of the three given, so (e) is correct.

3. Bill is going to make a 3-egg omelet, but little does he know that 2 of the eggs in the dozen eggs he just bought are poisoned. What is

the probability that Bill's omelet will be OK?

The sample space is $S = \{ \text{all possible sets of 3 eggs chosen from 12} \}$, and so $N(S) = \binom{12}{3} = 220$. The event of interest is $A = \{ \text{all possible sets of 3 eggs chosen from the 10 OK eggs} \}$, and so $N(A) = \binom{10}{3} = 120$. So, the probability that the omelet is OK is $120/220 = 0.545$.

4. Andy, Bill, and Claudia all hate each other. If they have 7 theater tickets in a row with 4 other people, and everyone sits down in a random order, what is the probability that none of Andy, Bill, and Claudia will be next to one another?

Let $S = \{ \text{all sets of three seats (not counting order) that these three can choose} \}$. Then $N(S) = \binom{7}{3} = 35$. Let $A = \{ \text{all sets of three seats (not counting order) with none adjacent} \}$. Then $N(A) = 10$, because we can list them:

$$A = \{(1, 3, 5), (1, 3, 6), (1, 3, 7), (1, 4, 6), (1, 4, 7), (1, 5, 7), (2, 4, 6), (2, 4, 7), (2, 5, 7), (3, 5, 7)\}.$$

So, the probability that none will be next to each other is $10/35 = 0.286$.

5. On my bureau, I have a dish containing 10 quarters, 3 nickels, 3 dimes, and 20 pennies. Rushing out the door, I grab 4 coins at random from the dish. What is the probability I will have one of each type of coin? What is the probability I will pick up two of one type of coin and two of another?

Let $S = \{ \text{all sets of 4 coins out of the 36 in the dish (order does not matter)} \}$. Then $N(S) = \binom{36}{4} = 58905$.

Let $A = \{ \text{all sets of 4 coins with one of each type (order does not matter)} \}$. Then $N(A) = (10)(3)(3)(20) = 1800$. So, the probability that I will choose one of each type of coin is $1800/58905 = 0.030$.

Let $B = \{ \text{all sets of 4 coins with two of one type and two of another type (order does not matter)} \}$. Then

$$\begin{aligned} N(B) &= \binom{10}{2} \binom{3}{2} + \binom{10}{2} \binom{3}{2} + \binom{10}{2} \binom{20}{2} + \binom{3}{2} \binom{3}{2} + \binom{3}{2} \binom{20}{2} + \binom{3}{2} \binom{20}{2} \\ &= (45)(3) + (45)(3) + (45)(190) + (3)(3) + (3)(190) + (3)(190) = 9969. \end{aligned}$$

So, the probability that I will choose two of one type of coin and two of another is $9969/58905 = 0.169$.

6. I have two jars of pickles, jar A with 70% spicy pickles and jar B with 20% spicy pickles. Unfortunately, the labels have fallen off the jars so I can no longer tell which is which.

(a) If I choose a jar at random, and then choose a pickle at random from that jar, what is the probability that I choose a spicy pickle?

Let $A = \{ \text{choose jar A} \}$ and $B = \{ \text{choose a spicy pickle} \}$. Then $P(A) = P(A') = 1/2$, and $P(B|A) = 0.7$ while $P(B|A') = 0.2$.

So, $P(B) = P(B \cap A) + P(B \cap A') = P(A)P(B|A) + P(A')P(B|A') = (0.5)(0.7) + (0.5)(0.2) = 0.45$.

(b) Let's say that I choose a jar at random, and then choose a pickle from that jar and find that it is spicy. Given that event, what is the probability that I chose jar A?

We are trying to compute $P(A|B)$. By Bayes' rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{(0.7)(0.5)}{0.45} = 0.778.$$

7. Seventy percent of the cars in Smallville own "the Club". Statistics show that 1% of cars with Clubs are stolen each year, as compared to 3% of cars without Clubs. If you find an abandoned stolen car outside your house in Smallville, what is the probability that it had a Club?

Let $A = \{ \text{car has a Club} \}$ and $B = \{ \text{car gets stolen this year} \}$. So, $P(A) = 0.7$, which means $P(A') = 0.3$. In addition, we are told $P(B|A) = 0.01$ while $P(B|A') = 0.03$.

So, $P(B) = P(B \cap A) + P(B \cap A') = P(A)P(B|A) + P(A')P(B|A') = (0.7)(0.01) + (0.3)(0.03) = 0.016$.

We are asked to compute $P(A|B)$, which, by Bayes' rule, equals

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{(0.7)(0.01)}{0.016} = 0.4375.$$

8. Ninety-five percent of hurricanes in the Atlantic miss Philadelphia. If we assume that each hurricane's path is independent of every other one, what is the probability that the next four hurricanes will miss Philadelphia?

If $A_1 = \{ \text{next hurricane misses Philly} \}$, $A_2 = \{ \text{2nd hurricane from now misses Philly} \}$, etc., then we are asked to compute $P(A_1 \cap A_2 \cap A_3 \cap A_4)$, which by independence equals $P(A_1)P(A_2)P(A_3)P(A_4)$, or $(0.95)(0.95)(0.95)(0.95) = 0.8145$.

9. Consider rolling three dice. Let A be the event that exactly two of the dice match, and let B be the event that you roll a 6 (total of the three dice). Are A and B independent?

Let $S = \{ \text{all rolls of three dice (with order chosen to matter)} \}$. Then, since each die has six possible results, $N(S) = 6^3 = 216$.

To compute $N(A)$, we observe that there are $\binom{3}{2} = 3$ ways of choosing a pair of dice to have equal values, then 6 choices for what that value will be, and then 5 remaining choices for the other die, so that $N(A) = (3)(6)(5) = 90$, so that $P(A) = 90/216 = 0.417$.

To compute $N(B)$, we just count up the possibilities— $(1,1,4)$, $(1,2,3)$, $(1,3,2)$, $(1,4,1)$, $(2,1,3)$, $(2,2,2)$, $(2,3,1)$, $(3,1,2)$, $(3,2,1)$, $(4,1,1)$ —to find $N(B) = 9$, so that $P(B) = 9/216 = 0.0417$.

Finally, to compute $N(A \cap B)$, we just count up all the rolls that add up to 6 and have exactly two matching dice— $(1,1,4)$, $(1,4,1)$, $(4,1,1)$ —to find $N(A \cap B) = 3$, so that $P(A \cap B) = 3/216 = 0.014$. Since this does not equal the product of $P(A)$ and $P(B)$, the events are not independent.