Math 116 Fall 2003—HW #5 Due: Fri. Oct. 10

Reading: Devore §4.1–4.4

Problems from Devore (wording shortened in some cases without affecting meaning):

For at least 5 of the problems involving computing integrals, you must work out the integrals by hand (for practice). For the rest, you may use your calculator or Mathematica; if you do, show the integral you are computing, and then say what technology you are using.

§4.1, # 1ab: Let X (the amount of time a book is checked out) have density

$$f(x) = \begin{cases} .5x & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $P(X \le 1)$ and $P(.5 \le X \le 1.5)$.

§4.1, # 4ab: Let X (the vibratory stress on a wind turbine blade) have density

$$f(x) = \begin{cases} \frac{x}{\theta^2} e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Verify that f is a legitimate pdf (i.e., has integral one), and, if $\theta = 100$, find the probability that X is at most 200, less than 200, and at least 200.

§4.1, # 8abde: Let Y (waiting time on two buses) have density

$$f(y) = \begin{cases} y/25 & 0 \le y < 5 \\ 2/5 - y/25 & 5 \le y \le 10 \\ 0 & \text{otherwise.} \end{cases}$$

Sketch a graph of f and verify its total integral is one. What is the proability that the total waiting time is at most 8 minutes? What is the probability that it is between 3 and 8 min?

§4.2, # 13 plus a median computation: Let X (headway between cars) have density

$$f(x) = \begin{cases} k/x^4 & x > 1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the value of k for which f is a legal density function. Obtain the cumulative distribution function and use it to determine the probability that the headway exceeds 2 seconds, and also the probability that the headway is between 2 and 3 sec. Obtain the mean value of the headway, the standard deviation of the headway, *and the median of the headway.* What is the probability that the headway is within 1 standard deviation of the mean?

§4.2, # 14ad: The depth of the bioturbation layer sediment has a uniform distribution on the interval $(7.5, 20)$. What are the mean and variance of depth? What is the probability that the observed depth is within 1 standard deviation of the mean? Within 2 standard deviations?

§4.2, # 21: An ecologist wishes to mark off a circular sampling region having radium 10 m. However, the radius of the resulting region is actually a random variable R with pdf

$$f(r) = \begin{cases} 0.75(1-(10-r)^2) & 9 \le r \le 11 \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected *area* of the resulting circular region?

§4.2, # 22cd plus a median computation: Let X (weekly demand for propane in 1000's of gallons) have density

$$f(x) = \begin{cases} 2\left(1-\frac{1}{x^2}\right) & 1 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

Compute $E(X)$, $V(X)$, *and the median of* X. If 1.5 thousand gallans are in stock at the beginning of the week, and no new supply is due in during the week, how much of the 1.5 thousand gallons is expected to be left at the end of the week?

§4.3, # 27bce: In each case, determine the value of the constant c that makes the probability statement correct: $P(0 \le Z \le c) = 0.291$, $P(c \le Z) = 0.1221$, and $P(c \le |Z|) = 0.016$, where Z has a standard normal distribution.

§4.3, # 31ab: Suppose the force acting on a column that helps to support a building is normally distributed with mean 15.0 kips and standard deviation 1.25 kips. What is the probability that the force is at most 18 kips? Is between 10 and 12 kips?

§4.3, #34: There are two machines for cutting corks. The first produces diameters normally distributed with mean 3 cm and standard deviation 0.1 cm. The second produces diameters normally distributed with mean 3.04 cm and standard deviation 0.02 cm. Acceptable corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork?

§4.3, # 37: The automatic opening device of a parachute has been designed to open when the parachute is 200 m above the ground. Suppose opening altitude is normal with mean 200 m and standard deviation 30 m. Equipment damage will occur if the parachute opens at an altitude of less than 100 m. What is the probability that there is equipment damage to at least one of five independently dropped parachutes?

§4.4, # 59: Let X (time between arrivals at a drive-up window) have exponential distribution with $\lambda = 1$. Compute the expected time between arrivals, the standard deviation of tiem between arrivals, $P(X \le 4)$, and $P(2 \le X \le 5)$.

§4.4, # 61a: The time until failure of fans used in diesel engines follows an exponential distribution with mean 25000 hours. What is the probability that a randomly selected fan will last at least 20000 hours? At most 30000 hours? Between 20000 and 30000 hours?