Math 116 Fall 2003—HW #3 Due: Fri. Sep. 19

**Reading**: Devore §2.4–2.5, 3.1–3.2

Problems from Devore (wording shortened in some cases without affecting meaning):

**§2.4 # 46**: Suppose a US adult male is randomly selected. Let $A$ be the event that he is over 6 feet tall, and let $B$ be the event that he is a professional basketball player. Estimate which is larger, $P(A|B)$ or $P(B|A)$, and explain.

**§2.4 # 49**: Consider a box with four 40-W bulbs, five 60-W bulbs, and six 75-W bulbs. If we choose two bulbs at random from this box, then, given that at least one of them is 75-W, what is the probability that both are 75-W? Given that at least of them is found to *not* be 75-W, what is the probability that both selected bulbs have the same wattage?

**§2.4, # 55**: Six people (three married couples) choose seats at random in a row of six seats.

(a) Compute the probability that Jim and Paula (one couple) sit together on the far left (event A) and John and Mary (another couple) sit together in the middle (event B). That is, compute $P(A \cap B)$ (not $P(A)$ and $P(B)$ separately).

(b) Given that John and Mary sit together in the middle, what is the probability that the two other husbands sit next to their wives?

(c) Given that John and Mary sit together (anywhere), what is the probability that all husbands sit next to their wives?

**§2.4, # 60**: Seventy percent of the aircraft that disappear are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator. Of the aircraft that are not discovered, 90% do not have an emergency locator. Suppose an aircraft has disappeared.

(a) If it has an emergency locator, what is the probability that it will not be discovered?

(b) If it does not have an emergency locator, what is the probability that it will be discovered?

**§2.5, # 72**: The proportion of blood types are 42% A, 10% B, 4 % AB, and 44 % O. If the blood types of two randomly selected people are independent, what is the probability that both are O? What is the probability that the blood types of two randomly selected people match?

**§2.5, # 80**: Independently roll two fair dice, one red, one green. Let $A$ be the event that the red die shows 3 dots, $B$ the event that the green die shows 4 dots, and $C$ the event that the total number of dots is 7. Are $A$ and $B$ pairwise independent? Are they mutually independent?

**§2.5, # 84**: (a) A lumber company has 10000 boards, but 2000 of them are damaged. Two boards are selected at random, one after the other. Let $A$ be the event that the first board is damaged, and $B$ the event that the second is damaged. Compute $P(A)$, $P(B)$, and $P(A \cap B)$.

(b) With $A$, $B$ independent and $P(A) = P(B) = 0.2$, what should $P(A \cap B)$ be? How much difference is there between this answer and $P(A \cap B)$ from part (a)? For purposes of computing $P(A \cap B)$, can we assume $A$, $B$ are independent to obtain essentially the correct probability?

(c) Suppose the lot consists of 10 rather than 10000 boards, with 2 of them damaged. Does the approximation of independence now yield approximately the correct answer of $P(A \cap B)$? What is the critical difference between the situation here and part (a)?

**§3.1, # 4**: Let $X$ be the number of nonzero digits in a randomly selected zip code. What are the possible values of $X$? Give three possible outcomes and their associated $X$ values.

**§3.2, # 12**: Suppose that a plane has 50 seats but 55 passengers have tickets. Define the random variable $Y$ to be the number of ticketed passengers that show up for the flight. The probability mass function of $Y$ appears below:

| $y$ | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $p(y)$ | 0.05 | 0.10 | 0.12 | 0.14 | 0.25 | 0.17 | 0.06 | 0.05 | 0.03 | 0.02 | 0.01 |

(a) What is the prob. that the flight will accommodate all ticketed passengers who show up?

(b) What is the prob. that not all ticketed passengers who show up can be accommodated?

(c) If you are the first person on the standby list, what is the probability that you will be able to take the flight? What is this probability if you are third person on the standby list?

**§3.2, # 15**: Suppose a computer manufacturer receives computer boards in lots of five. Two boards are selected from each lot for inspection. We can represent possible outcomes of the selection process by pairs, e.g., $(1, 2)$ represents selecting boards 1 and 2.

(a) List the ten different possible outcomes (order within the pair doesn't matter)

(b) Suppose that boards 1 and 2 are the only defective boards in the lot. Let $X$ be the number of defective boards observed among those inspected. Find the probability distribution of $X$.

(c) Let $F(x)$ denote the cdf of $X$. First determine $F(0) = P(X \leq 0)$, $F(1)$, and $F(2)$, and then obtain $F(x)$ for all values of $x$.

**§3.2, # 18**: Two fair six-sided dice are tossed independently. Let $M$ be the maximum of the two tosses. (a) What is the pmf of $M$? (b) Determine the cdf of $M$ and graph it.

**§3.2, # 25ac**: Alvie lives at 0 in the accompanying diagram and has four friends who live at A, B, C, and D. One day, Alvie decides to go visiting, so he tosses a fair coin twice to decide which of the four to visit. Once at a friend's house, he will either return home or else proceed to one of the two adjacent houses (such as 0, A, or C when at B), with each of the three possibilities having probability 1/3. In this way, Alvie continues to visit friends until he returns home.

(a) Let $X$ be the number of times Alvie visits a friend. Derive the pmf of $X$. Explain.

(c) Suppose that female friends live at A and C and male friends at B and D. Let $Z$ be the number of visits to female friends. What is the pmf of $Z$? Explain.